Asymptotic Stability of a Walking Cycle for a Biped Robot with Knees and Torso

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ABSTRACT

This paper proposes an asymptotically stabilizing feedback controller for an under actuated, walking 7-DOF biped robot. The controller is inspired by analytical work that was carried out previously on a 5-DOF walker. A key features is that a walking motion is created through the design of holonomic constraints that are imposed via feedback control instead of through the tracking of pre-planned trajectories. This theoretically motivated control design method is shown to yield a control law that will be practically implementable on a robot that is under construction.

1. INTRODUCTION

This paper considers a five-link planar biped robot consisting of a torso and two legs with knees (see Figure 1). The robot has four independent actuators: the axis between the torso and each thigh is actuated as is the axis of each knee. The dynamic model of this 7 DOF biped in conjunction with an impact model can be expressed as a nonlinear system with impulse effects [12]. This paper concentrates on issues related to the automatic control of this walking robot and is a first step toward the formal proof of the asymptotic stability of the walking motion of an under actuated biped robot with a torso and two knees. The work presented here is a natural continuation of [5,6] where the asymptotic stability of the walking motion of a robot with a torso and no knees was fully proved.

A feedback controller is given for the biped robot. The goal of the controller design is to induce an asymptotically stable walking cycle. At its most basic level, walking consists of posture control, that is, maintaining the torso in a semi-erect position, height control, that is, keeping the hips at a relatively constant height above the walking surface, and swing leg advancement, that is, causing the swing leg to come from behind the stance leg, pass it by a certain amount, and prepare for contact with the ground. As in [5,6], these objectives are built into a set of holonomic constraints on the generalized position coordinates, q. The objective of the feedback controller is then to impose these constraints on the system, by way of driving the constraints to zero. Of course, due to the perturbations from the impacts with the walking surface, the controller is unable in general to force the constraints to approach zero and remain at zero for all time. A general means of trying to "overcome" this can be observed in the literature: for experimental as well as simulation based studies, the feedback "gains" appear to be *universally chosen large enough so that the time constant for driving the constraints to zero is much less than the time interval of a single step.*

On the other hand, the work in [5,6] for the biped model without knees achieved "rapid" convergence of the design constraints to zero without using high gain control. Continuous, but non-Lipschitz continuous, feedbacks were employed [1]. With such feedbacks, the trajectories of the stiff-legged biped were made to converge to a certain zero dynamics manifold, Z, in finite-time, while using a bounded input. By adjusting the time of convergence to be less than the time of a step, the image of the Poincaré map was made to lie in a one dimensional submanifold, denoted $S \cap Z$. For the stiff-legged model, it was then possible to introduce a map λ that was essentially the restriction of the Poincaré return map, P, to $S \cap Z$, and to prove that the existence and stability properties of orbits of the closed-loop system can be established through the analysis of fixed points of λ. For a system with one less actuator than degrees of freedom, the dimension of $S \cap Z$ is one. *This same method is illustrated here on the more general biped model.*

This paper does not provide a formal proof of the asymptotic stability for a biped robot with torso and knees. Rather, it shows that the theoretically motivated controller structure of [5,6] can be successfully applied to more complicated models. To the extent possible, the key hypotheses and tools which are needed to prove asymptotic stability are stated. Simulations results of the gait of the 7 DOF biped are given.

The model employed is based upon a biped robot that is currently under construction (Site in French - www-lag.ensieg.inpg.fr/recherche/cser/PRC-Bipedes/Prototype/rabbit.html). The torques that are demanded by the controller developed here meet the constraints of the robot under construction.

2. ROBOT MODEL

The robot considered is planar and bipedal. It consists of a torso, hips and two legs of equal length, with knees but no ankles (see Figure 1). It thus has 7 degrees of freedom. A torque is applied between each leg and the torso, and a torque is applied at each knee. It is assumed that the walking cycle takes place in the sagittal plane and consists of successive phases of single support.

The complete model of the biped robot consists of two parts: the differential equations describing the dynamics of the robot during the swing phase (these equations are derived using the method of Lagrange [10]), and an impulse model of the contact event (the impact between the swing leg and the ground is modeled as a contact between two rigid bodies [7]). The contact between the stance leg and the ground is modeled as a pivot. As in [5,6], the complete model can be expressed as a nonlinear system with impulse effects [12].

2.1 Swing phase model

The dynamic model of the robot between successive impacts is derived from the Lagrange formalism

$$D(q)\cdot\ddot{q} + C(q,\dot{q})\cdot\dot{q} + G(q) = B\cdot u \qquad (1)$$

with $q=(q_1,q_{31},q_{41},q_{32},q_{42})'$ (see Figure 1) and $u=(u_1,u_2,u_3,u_4)'$. The torques u_1, u_2, u_3 and u_4 are applied between the torso and the stance leg, the torso and the swing leg, at the knee of the stance leg and at the knee of the swing leg, respectively. Then, the model can be written in state space form by defining

$$\dot{x} := \frac{d}{dt}\begin{bmatrix} q \\ \omega \end{bmatrix} = \begin{bmatrix} \omega \\ D^{-1}(q)\cdot(-C(q,\omega)\cdot\omega - G(q) - B\cdot u) \end{bmatrix} =: f(x) + g(x)\cdot u \tag{2}$$

2.2 Impact model

The impact between the swing leg and the ground is modeled as a contact between two rigid bodies. The contact model requires the full seven degrees of freedom of the robot. Let us add Cartesian coordinates (x_1,z_1) to the end of the stance leg (see Figure 1). One then obtains the following extended model

$$D_e(q_e)\cdot\ddot{q}_e + C_e(q_e,\dot{q}_e)\cdot\dot{q}_e + G_e(q_e) = B_e\cdot u + \delta F_{ext} \tag{3}$$

with $q_e=(q_1,q_{31},q_{41},q_{32},q_{42},x_1,z_1)'$. δF_{ext} represents the external forces acting on the robot at the contact point. The basic hypotheses are

- The contact of the swing leg with the ground results in no rebound and no slipping of the swing leg.
- At the moment of impact, the stance leg lifts from the ground without interaction.
- The impact is instantaneous.
- The external forces during the impact can be represented by impulses.
- The impulsive forces may result in an instantaneous change in the velocities, but there is no instantaneous change in the positions.
- The torques supplied by the actuators are not impulsional.

From these hypotheses, the angular momentum is conserved. One deduces

$$D_e(q_e)\cdot(\dot{q}_e^+ - \dot{q}_e^-) = F_{ext} \tag{4}$$

where F_{ext} is the result of the contact impulse forces. $\dot{q}_e^+$ (resp. $\dot{q}_e^-$) is the velocity just after (resp. before) impact. An additional set of two equations is obtained by supposing that the swing leg does not rebound nor slip at impact (note that these two equations are needed to be able to solve for the impact forces and the post-impact velocities, since there are seven unknowns). This yields

$$E(q_e)\cdot\dot{q}_e^+ = 0 \tag{5}$$

The result of solving (4) and (5) yields an expression for $\dot{q}_e^+$ in term of $\dot{q}_e^-$. The final result is an expression for $x^+ := (q^+,\omega^+)$ (state value just after the impact) in terms of $x^- := (q^-,\omega^-)$ (state value just before the impact), which is expressed as

$$x^+ = \Delta(x^-) \tag{6}$$

2.3 Nonlinear system with impulse effects

The overall biped robot model can be expressed as a nonlinear system with impulse effects

$$
\begin{aligned}
&\dot{x} = f(x) + g(x)\cdot u \qquad && x^- \notin S \\
&x^+ = \Delta(x^-) && x^- \in S
\end{aligned}
\tag{7}
$$

Where, letting (x_2,z_2) denote the Cartesian coordinates of the end of the swing leg (see Figure 1), $S = \{(q,\omega) \in \mathrm{X} \mid z_2 = 0\}$ and $\mathrm{X} := \{(q',\omega')' \mid q \in (-\pi,\pi)^5, \omega \in \Re^5\}$.

3. ASYMPTOTICALLY STABILIZING CONTROLLER

This section develops the extension of the controller of [5,6] for the 5 link biped with knees.

3.1 Output definitions

A set of outputs is defined as follows

$$
\begin{aligned}
y_1 &= k_1 \cdot (q_1 - q_{1d}) \\
y_2 &= k_2 \cdot (d_1 + d_2) \\
y_3 &= k_{31} \cdot (z_H - z_{Hd}) + k_{32} \cdot (q_{41} - q_{31} - q_{Kd}) \\
y_4 &= k_4 \cdot (z_2 - z_{2d}(d_1))
\end{aligned}
\tag{8}
$$

The coordinates of the hips, *(x_H,z_H)*, and the “foot” of the swing leg, *(x_2,z_2)* are expressed in the coordinate frame of the foot of the stance leg *(x_1,z_1)* (see Figure 1).

The output y_1 is chosen to maintain the angle of the torso at a desired constant value, say q_{1d}. The output y_2 ensures the advancement of the hips while the swing leg goes from behind the stance leg to in front of it (see Figure 1 for representation of d_1 and d_2). The output y_3 controls the hip height and the flexing of the stance leg knee (z_{Hd} and q_{Kd} are constant values). The output y_4 controls the trajectory of the end of the swing leg; the desired trajectory z_{2d} is defined as a second order polynomial of d_1 such that $d_1 \in [-sld/2, sld/2]$, where *sld* is the desired step length, z_{2MAX} is the maximum desired value of z_2 over a step and

$$
z_{2d}(-sld/2) = 0 \qquad z_{2d}(0) = z_{2MAX} \qquad z_{2d}(sld/2) = 0 \tag{9}
$$

The gains k_1, k_2, k_{31}, k_{41} and k_4 are constant values to be chosen later. Thus, with the same notation as in (7), the output vector reads as

$$
y = \begin{bmatrix} k_1 \cdot (q_1 - q_{1d}) \\ k_2 \cdot (d_1 + d_2) \\ k_{31} \cdot (z_H - z_{Hd}) + k_{32} \cdot (q_{41} - q_{31} - q_{Kd}) \\ k_4 \cdot (z_2 - z_{2d}(d_1)) \end{bmatrix} := h(q) = \begin{bmatrix} h_1(q) \\ h_2(q) \\ h_3(q) \\ h_4(q) \end{bmatrix} \tag{10}
$$

3.2 Controller synthesis

The control objective is to drive the outputs (10) to zero. Since the biped model comes from the second order model (2) and the outputs (10) only depend on functions of the generalized positions, q, the relative degree of each output component is either two or infinite. Direct calculation yields

$$\ddot{y} = L_f^2 h(x) + L_g L_f h(x) \cdot u \tag{11}$$

For the purposes of this conference paper, it is supposed that the matrix $L_g L_f h$ is invertible on the region of interest. This has in fact been confirmed by numerical computations. Define

$$v = L_f^2 h(x) + L_g L_f h(x) \cdot u \tag{12}$$

The next step is to define a continuous feedback $v = v(y, \dot{y})$ on (11) so that the four pairs of double integrators $\ddot{y} = v$ are (globally) finite-time stabilized. The feedback solution used here comes from [1].

Lemma 1 [1] *Consider the double integrator*

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= v \end{aligned} \tag{13}$$

with $v \in \Re$. *Then, with* $0 < \alpha < 1$, *the feedback*

$$\psi_\alpha(x_1, x_2) := -\mathrm{sign}(x_2) \cdot |x_2|^\alpha - \mathrm{sign}(\phi_\alpha(x_1, x_2)) \cdot |\phi_\alpha(x_1, x_2)|^{\frac{\alpha}{2-\alpha}} \tag{14}$$

where the function $\phi_\alpha(x_1, x_2)$ *is defined by*

$$\phi_\alpha(x_1, x_2) = x_1 + \frac{1}{2-\alpha} \cdot \mathrm{sign}(x_2) \cdot |x_2|^{2-\alpha} \tag{15}$$

satisfies the following

- *v is continuous,*
- *the origin of (13) in closed-loop with (14) is globally finite-time stable,*
- *the settling time function T_{set} depends continuously on the initial condition.*

♦

In the case of the biped robot, and with the notation of [1], a finite-time-stabilizing controller is given by

$$v = \psi(y, \dot{y}) := \begin{bmatrix} \psi_1(y_1, \varepsilon \cdot \dot{y}_1) \\ \psi_2(y_2, \varepsilon \cdot \dot{y}_2) \\ \psi_3(y_3, \varepsilon \cdot \dot{y}_3) \\ \psi_4(y_4, \varepsilon \cdot \dot{y}_4) \end{bmatrix} \tag{16}$$

where each function $\psi_i(y_i, \dot{y}_i)$ ($1 \le i \le 4$) has the same form as (14). The reader is referred to [1,5,6] for more theoretical details. Then, define a feedback on (7) by

$$u(x) := (L_g L_f h(x))^{-1} \cdot (\psi(h(x), L_f h(x)) - L_f^2 h(x)) \qquad (17)$$

3.3 Checking asymptotic stability

The asymptotic stability of the walking cycle of the biped robot (7) under the control law (17) should be verified by analysing fixed points of the associated Poincaré map. A key point developed in [5,6] is that the special feedback controllers used here allow the asymptotic stability of a walking motion to be checked on the basis of a reduced Poincaré map. Moreover, a convenient numerical procedure can be given to compute the reduced map. The numerical procedure is recalled here; for its theoretical underpinnings, the reader is referred to [5,6].

Let $v_H^- = \dot{x}_H^-$ denote the horizontal velocity of the hips just before impact. Define the function σ by

$$\sigma(v_H^-) := (q_1^d \quad q_{31}^- \quad q_{32}^- \quad q_{41}^- \quad q_{42}^- \quad 0 \quad \dot{q}_{31}^- \quad \dot{q}_{32}^- \quad \dot{q}_{41}^- \quad \dot{q}_{42}^-)' \qquad (18)$$

where all the state components are computed by supposing that the outputs (10) are identically zero, and $v_H^- = \dot{x}_H^-$.

Numerical Procedure

- For a point $v_H^- > 0$ (which corresponds to the robot walking from left to right), compute $x^- := \sigma(v_H^-)$.
- Apply the impact model to x^-, in order to compute $x^+ = \Delta(x^-)$, the state of the biped just after impact.
- Use x^+ as the initial condition of the robot in closed loop with the feedback controller, and simulate until one of the following happens
    - There exists a time $T > 0$ where $z_2(T)=0$, then, if T is greater than the settling time of the controller (i.e. the outputs are identically zero), define $\lambda(v_H^-) = v_H(T)$; else, $\lambda(v_H^-)$ is undefined at this point.
    - There does not exist a time $T > 0$ such that $z_2(T)=0$, in this case, $\lambda(v_H^-)$ is undefined at this point.

Analysis Result

The essence of the theory developed in [5,6] is that a fixed point of λ, i.e $\lambda(v_H^{-*}) = v_H(T) = v_H^{-*}$, corresponds to a periodic orbit of the biped model. Moreover, if for v_H^- near v_H^{-*}, $v_H^- < v_H^{-*} \Rightarrow \lambda(v_H^-) > v_H^-$, then the fixed point corresponds to an asymptotically stable periodic orbit, or in other words, an asymptotically stable walking cycle.

4. SIMULATIONS

Consider the biped robot model (7) with the following parameter values

$$M_T = 20\text{kg} \quad M_3 = 6\text{kg} \quad M_4 = 6\text{kg} \quad l_T = 0.625\text{m} \quad l_3 = 0.4\text{m} \quad l_4 = 0.4\text{m} \qquad (19)$$

corresponding to the mass of the torso, the mass of the femur, the mass of the tibia, the length of the torso, the length of the femur and the length of the tibia, respectively. For the outputs defined in the previous section, the invertibility of the decoupling matrix can be numerically verified. Suppose that q_{1d}=π/30rad, z_{Hd}=0.74m, q_{Kd}=π/18rad, sld=0.5m, z_{2MAX}=1cm, k_1=0.25, k_2=k_3=k_4=100 and v_H^- =1.05ms^{-1}. In the feedback (17), suppose

$$\psi(x) := \begin{bmatrix} \frac{1}{9\varepsilon^2}\cdot\psi_1(y_1,\varepsilon\cdot\dot{y}_1) \\ \frac{1}{2.25\varepsilon^2}\cdot\psi_2(y_2,\varepsilon\cdot\dot{y}_2) \\ \frac{1}{\varepsilon^2}\cdot\psi_3(y_3,\varepsilon\cdot\dot{y}_3) \\ \frac{25}{\varepsilon^2}\cdot\psi_4(y_4,\varepsilon\cdot\dot{y}_4) \end{bmatrix} \quad (20)$$

with $\varepsilon=0.075$ and $\alpha=0.9$ (the parameter $\varepsilon> 0$ allows the settling time of the controller to be adjusted.) All the previous parameter and coefficient values have been chosen to minimize the required torques, a practical constraint.

Figures 2-7 present representative simulation results over a few walking cycles near the stable orbit. Figures 2-3 display the state and the applied torques over a few walking cycles (about five steps); note that the peak torque magnitude is about 45 Nm. Figure 4 displays the outputs, which go to zero prior to impact. Figure 5 displays the function λ; it also displayed the function $\lambda(v_H^-)-v_H^-$ which represents the change in the hip velocity over successive cycles, from just before one impact to just before the next one. A fixed point, corresponding to an asymptotically stable walking cycle, occurs at approximately 1.16 ms^{-1}. Figures 6-7 display the positions and velocities of the hips and the foot of the swing leg.

5. REFERENCES

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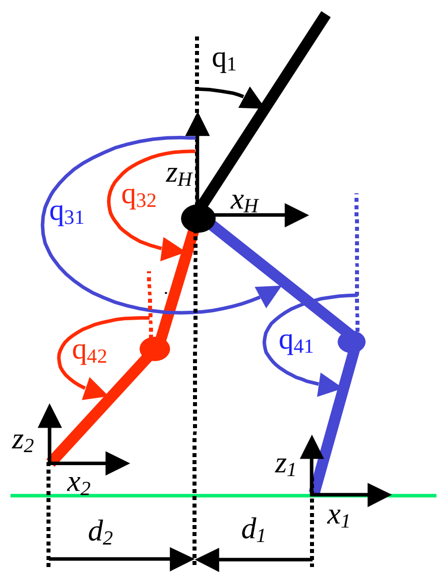

Fig. 1: Schematic indicating the definition of the coordinates of the biped robot.

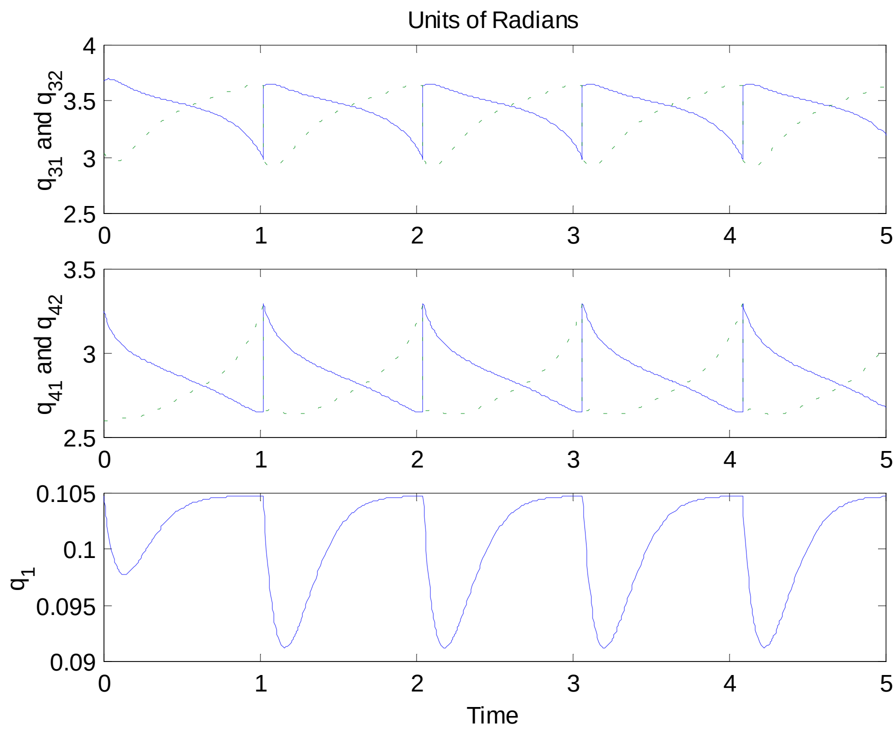

Fig. 2: Plot of joint angles versus time; unit of radian

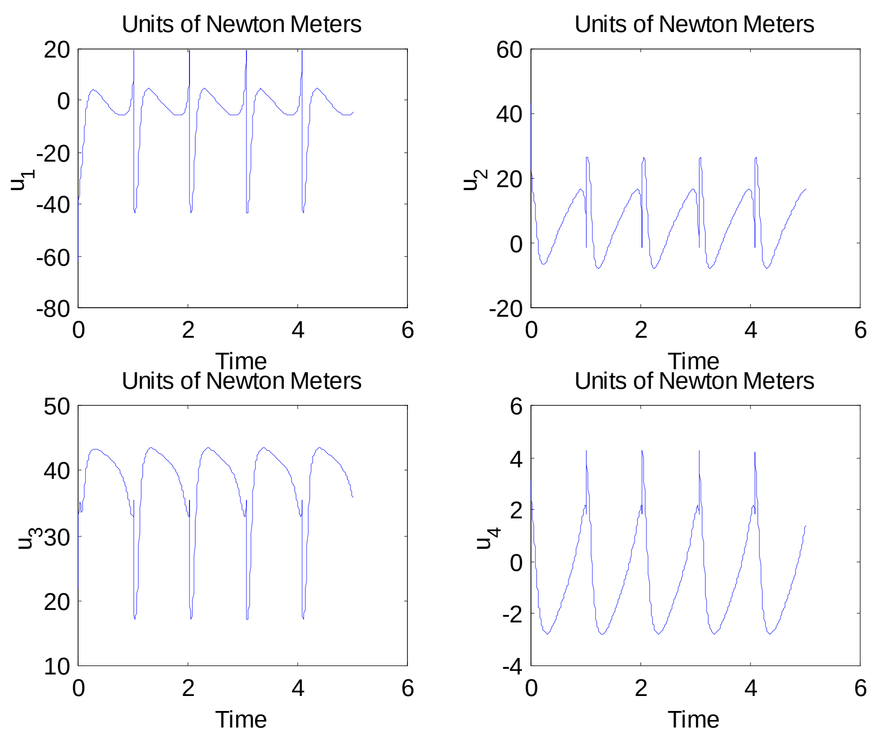

Fig. 3: Plot of applied torques versus time; unit of Newton-Meter.

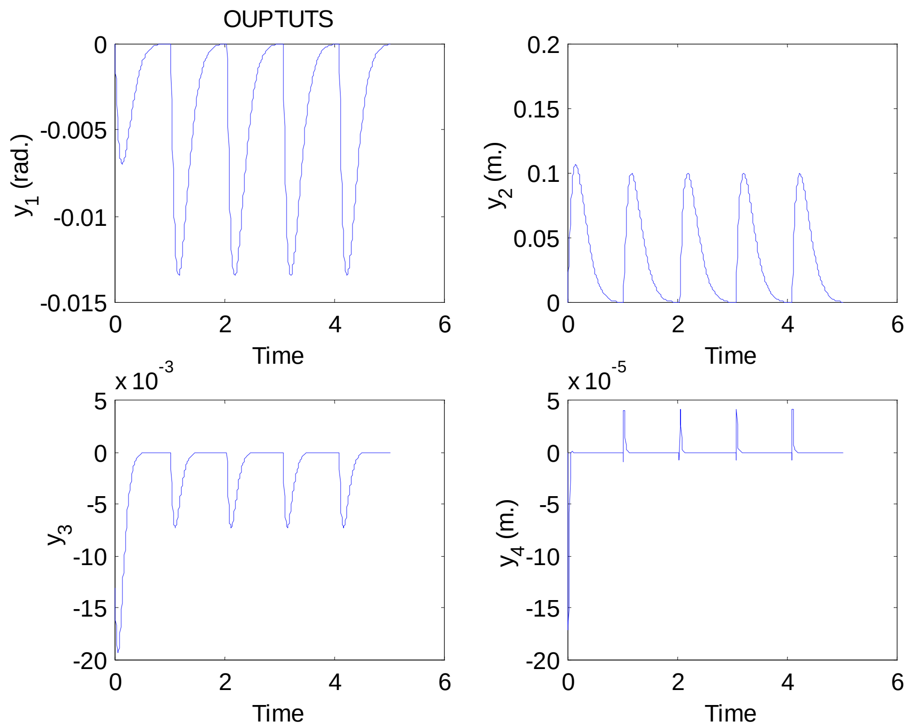

Fig. 4: Plot of outputs versus time.

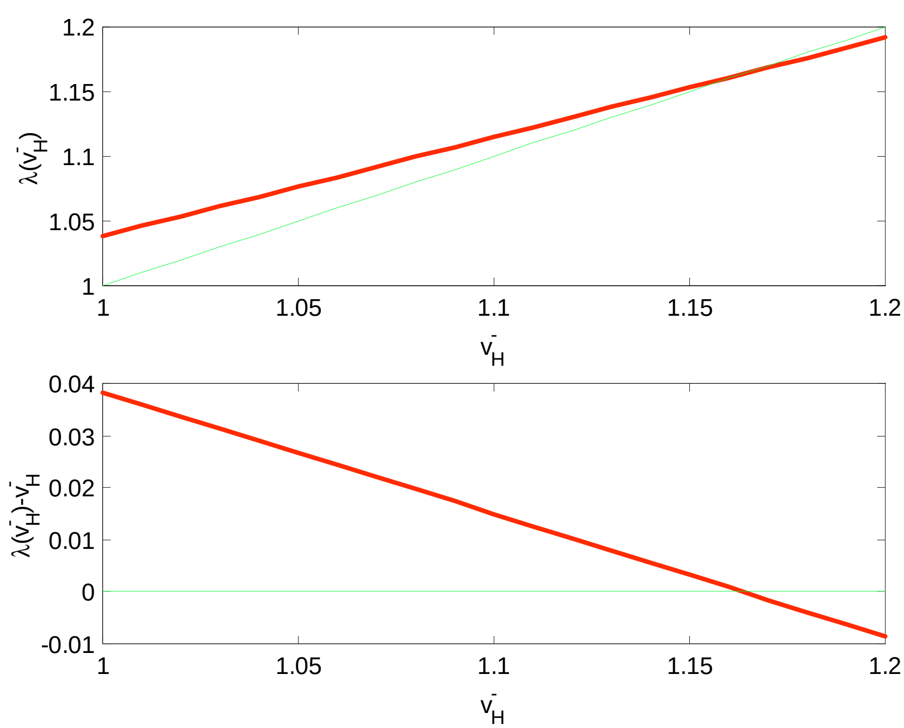

Fig. 5: The top graph presents the function λ (bold line) and the identify function (thin line); the bottom graph presents the function $\lambda - v_H^-$ (bold line) and the zero line (thin line).

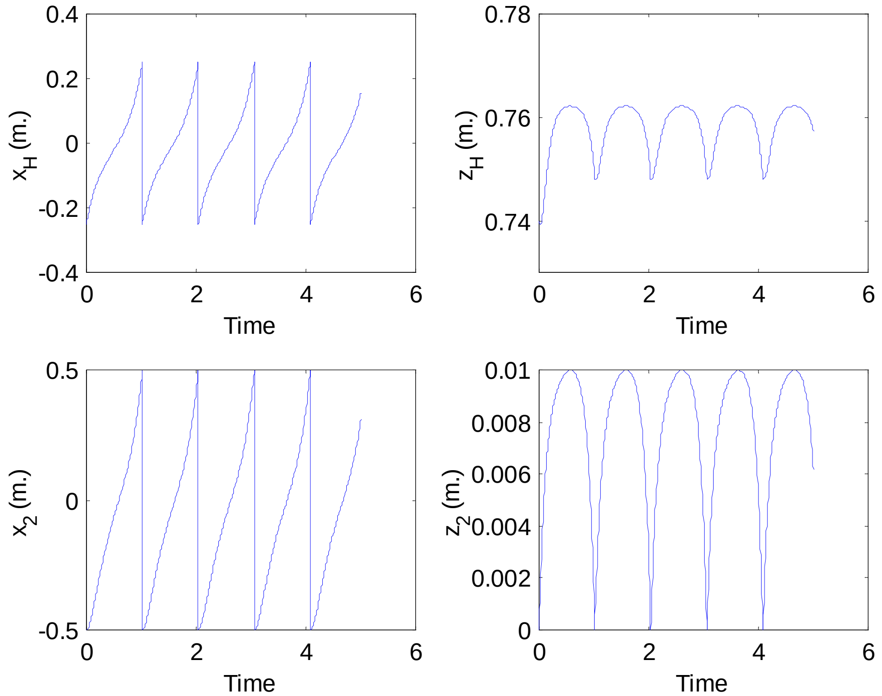

Fig. 6: Plot of positions of the hips and the swing leg end versus time; unit of meter

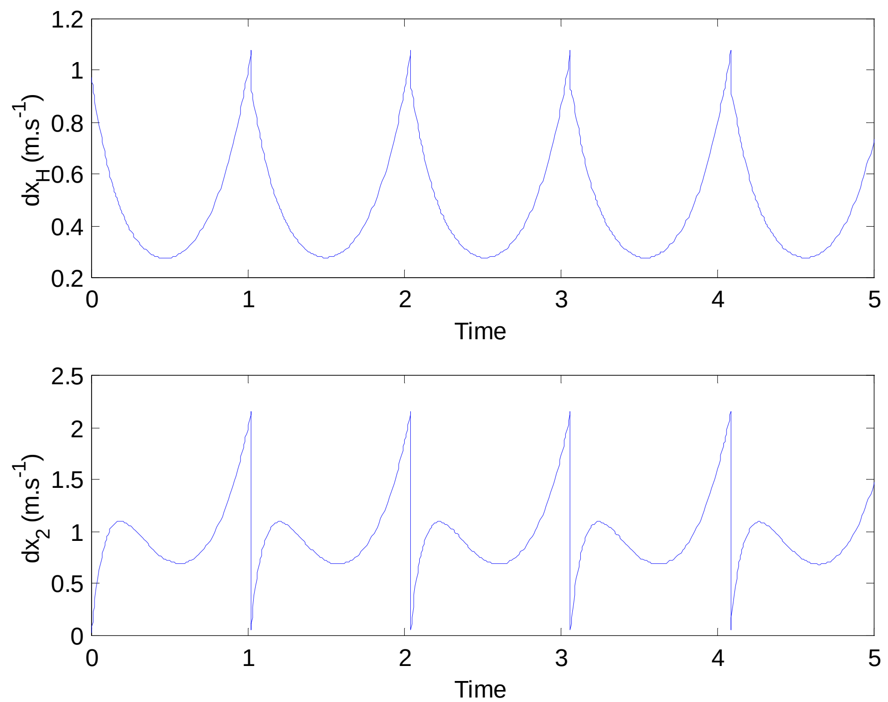

Fig. 7: Plot of the horizontal velocities of the hips and the swing leg end versus time; unit of meter/second.