Approximate Dynamic Programming Solutions for Lean Burn Engine Aftertreatment

Jun-Mo Kang[1], Ilya Kolmanovsky[2] and J. W. Grizzle[3]

Abstract

The competition to deliver fuel efficient and environmentally friendly vehicles is driving the automotive industry to consider ever more complex powertrain systems. Adequate performance of these new highly interactive systems can no longer be obtained through traditional approaches, which are intensive in hardware use and final control software calibration. This paper explores the use of dynamic programming to make model-based design decisions for a lean burn, direct injection spark ignition engine, in combination with a three way catalyst and lean NOx trap aftertreatment system. The primary contribution is the development of a very rapid method to evaluate the tradeoffs in fuel economy and emissions for this novel powertrain system, as a function of design parameters and controller structure, over a standard emission test cycle.

1 Introduction

Designing a powertrain system to meet drivability, fuel economy and emissions performance requirements is a complicated task. There are many tradeoffs to be analyzed in terms of which components to use, such as lean burn technology versus classical components, characteristics of individual components, such as size or temperature operating range, and the control policies to be employed. In addition, there are tradeoffs to be analyzed among the performance metrics themselves, such as emissions versus fuel economy. In the past, most of the powertrain design decisions were on the basis of hardware, that is, on the basis of laboriously assembling and evaluating many possible system configurations. Today, the time-line for vehicle design is constantly shrinking, the number of possible powertrain configurations is expanding, and the cost of doing hardware evaluations is growing. It is simply no longer feasible, economically, nor time-wise, to make all (or even most) of the design decisions on the basis of hardware alone. More and more of the decisions must be made upon the basis of mathematical models and analysis.

This paper will describe the use of dynamic programming to assist in making powertrain design decisions on the basis of component models. The specific technology configuration analyzed here involves a direct injection spark ignition (DISI) engine. In this type of engine, fuel is directly injected into the combustion chamber during the compression stroke, and the highly concentrated fuel around the spark plug and extensive air motion enables combustion of an overall lean mixture (the shape of the piston is specially designed to enhance air motion (swirl or tumble), and it is further enhanced in the compression process) [7]. The DISI engine studied here can operate in either homogeneous or stratified mode. In stratified mode, the engine can operate at air-to-fuel ratios up to 40:1. At such lean air-to-fuel ratios, the three-way catalyst (TWC) can effectively convert CO and HC in the exhaust gas to CO_2 and H_2O. However this is not the case for NO_x, and as a result, it is necessary to develop a technique for NO_x removal. The current technique is to place a Lean NO_x Trap (LNT) after the TWC in the exhaust system. NO_x is trapped in the LNT while the engine operates at a lean condition. By periodically operating the engine at a rich condition (in homogeneous mode), the trapped NO_x is purged and converted to N_2 by reductants such as CO and H_2 [1, 2]. It follows that both emissions and fuel consumption strongly depend upon the duration and frequency of the purging mode (rich operation of the engine), and obviously the control strategy for purging the LNT should be well optimized to achieve high fuel economy and low NO_x emissions.

Section 2 sets up the fuel economy versus emissions tradeoff problem in the context of a dynamic programming problem. Section 3 explores solution times using standard state space discretization methods; it will be seen that computation times are too long for the engineer to do case study analysis. Section 4 introduces a method for rapidly generating suboptimal solutions; a simple case is analyzed to show that the method can potentially produce near optimal solutions. The computation time is reduced by a factor of twenty. Section 5 points out how the computation speed can be fur-

[1]Jun-Mo Kang is a Ph.D. candidate in Electrical Engineering and Computer Science Department, University of Michigan, Email: junmo@eecs.umich.edu.

[2]Ilya Kolmanovsky is with the Ford Research Laboratory, Dearborn, MI 48121-2053, Email: ikolmano@ford.com.

[3]J. W. Grizzle is with the Control Systems Laboratory, Electrical Engineering and Computer Science Department, University of Michigan, Ann Arbor, MI 48109-2122,Tel: (734)-763-3598, FAX: (734)-763-8041, Email: grizzle@umich.edu.

ther enhanced through vectorization of the MATLAB code. Section 6 looks at several case studies using this optimization tool.

2 Mathematical Problem Formulation

A finite horizon optimization problem for determining a control strategy of the combined DISI engine and exhaust aftertreatment system depicted in Figure 1 is posed in this section. The LNT is a dynamic device

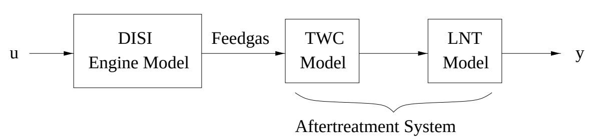

Figure 1: Complete model for emission system.

in the sense that its capability to trap NO_x dynamically changes until it reaches saturation, and similarly, the TWC dynamically stores and releases oxygen in the feedgas. Control-oriented, temperature-dependent, dynamic models of the TWC and LNT have been developed in [5, 6] and [4], respectively. A control-oriented model of a 1.8L DISI engine is given in [8]. Due to space limitations, these models are not reviewed here.

A model of the combined engine and emissions systems, discretized for numerical optimization, can be expressed as:

$$\begin{aligned} x(k+1) &= f(x(k), u(k)) \\ y(k) &= h(x(k), u(k)), \end{aligned} \tag{1}$$

where $u(k)$ is the vector of engine input parameters such as throttle position, fuel mass flow rate, spark timing, and EGR rate, $x(k)$ is the vector of states of the overall system and $y(k)$ is the tailpipe NO_x emissions out of the LNT.

The objective of the study is to evaluate the tradeoff in fuel economy and NOx emissions[1]. The instantaneous cost is chosen as a weighted sum:

$$\begin{aligned} g(y(k), u(k)) &= \text{fuel}(k) + \mu \cdot NO_x(k) \\ &= \text{fuel}(k) + \mu \cdot y(k). \end{aligned} \tag{2}$$

In general, the emission performance of a vehicle is evaluated through a specific drive test cycle such as the US FTP cycle, or the European Drive Cycle. Then the objective is to find the optimal control input, $u(k)$, that minimizes the cost functional

$$J(x) = \min_{u \in U} \sum_{k=0}^{N-1} g(y(k), u(k))$$

$$= \min_{u \in U} \sum_{k=0}^{N-1} \bar{g}(x(k), u(k)) \tag{3}$$

where U represent constraints for u imposed by meeting the speed and load demands of the specific drive cycle, plus things like intake manifold pressure being positive and not exceeding one atmosphere (unless boosted); N is the time length of the drive cycle. The cost (3) represents the cumulative weighted sum of fuel consumption and NOx over the drive cycle. The objective will be to

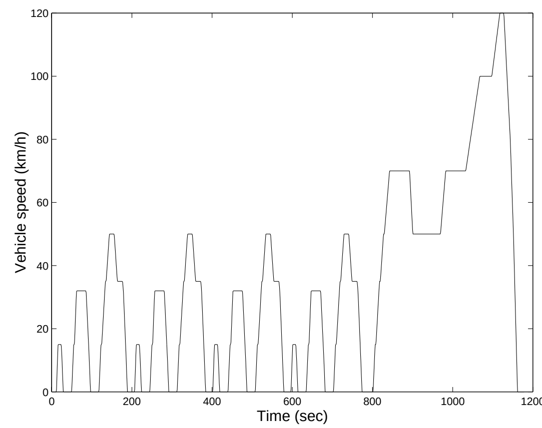

Figure 2: European drive cycle for emissions evaluation.

minimize the cost function (3), for a range of μ. This will provide information on the sensitivity of fuel economy to NOx emission levels, and is more useful than just knowing the best fuel economy for a given emissions constraint. A systematic solution to the above problem can be determined recursively via Bellman's Dynamic Programming (DP) Algorithm [3] as follows:

Step $N-1$:

$$J_{N-1}(x(N-1)) := \min_{u(N-1) \in \mathcal{U}(N-1)} \left[g(y(N-1), u(N-1)) \right]$$

$$= \min_{u(N-1) \in \mathcal{U}(N-1)} \left[\bar{g}(x(N-1), u(N-1)) \right], \tag{4}$$

where $\bar{g}(x, u) := g(h(x, u), u)$.

Step k, for $N-1 > k \geq 0$:

$$J_k(x(k)) := \min_{u(k) \in \mathcal{U}(k)} \left[\bar{g}(x(k), u(k)) + J_{k+1}(f(x(k), u(k))) \right] \tag{5}$$

End.

The optimal control policy is then any minimizer of (5).

3 Numerical Dynamic Programming

It is well-known that the computation time of the DP algorithm is exponential in the number of states. For

[1] Since a DISI engine is mostly operated in a lean mode, it is felt that CO and HC levels should not be problem. The only exception would occur if the LNT is purged too often, which would also show up as a fuel economy penalty

this reason, it is important to make a judicious choice of the complexity of the dynamic models involved. In addition, it is standard to discretize the state space with a grid for numerical computation.

3.1 Model Simplification

The NOx fill time of the LNT is on the order of thirty seconds to one minute, and its purge time is on the order of a few seconds. The key dynamics of the TWC in this context is its oxygen storage capability; the TWC oxygen fill time under very lean conditions can be as short as a second, while its time to empty is several seconds. The time constants of the temperature dynamics of the TWC and LNT are on the order of 10 seconds each. Finally, the most important dynamics of the engine is the intake manifold filling model, which has a time constant on the order of a 100 milli-seconds. It is concluded from this that the dominant dynamics are in the emission system, and the engine can be treated as a static device delivering torque and exhaust feedgas (emissions concentrations, flow rates, temperature) as a function of throttle position, fuel flow, spark and EGR.

3.2 Standard State Space Discretization

The standard method to convert a Dynamic Program [3, 10] into a finite computation problem is to use state space quantization and function interpolation [3]. The state space is quantized into a finite grid

$$x \in \{\eta_1, \eta_2, \ldots, \eta_L\} \tag{6}$$

At each step of the DP algorithm, the function $J_k(x(k))$ is determined at a finite number of points, $\{\eta_1, \ldots, \eta_L\}$. The function $J_k(x(k))$ at an arbitrary point is then approximated by linear interpolation. In general, a successful approximation of this type of discretization depends upon 'consistency'. This means that a solution closer to a continuous optimal solution can be achieved as the discretization becomes finer [3], which in turn imposes increased computational burden.

Spatial discretization yields the following general step of the DP algorithm:

Step k, for $N-1 > k \geq 0$, and for $1 \leq i \leq L$:

$$J_k(\eta_i) := \min_{u(k)\in\mathcal{U}(k)} \left[\bar{g}(\eta_i, u(k)) + \hat{J}_{k+1}(f(\eta_i, u(k)))\right], \tag{7}$$

where $\hat{J}_k$ is defined by interpolating $\{J_k(\eta_1), \ldots, J_k(\eta_L)\}$.

To check the computational complexity, the above program was setup in MATLAB, with a static TWC model and a one state (NOx storage level), temperature-dependent, LNT model as the exhaust aftertreatment system. The state was discretized as $0.15 \times \{0, 0.05, \ldots, 0.45, 0.5, 0.7, 0.9, 1\}$ (the maximum trap capacity of the LNT was set to be 0.15 g). The European Drive Cycle (Euro-cycle), shown in Figure 2, was used. The cycle was sampled at the rate of one second. The engine speed and torque required to follow the cycle at each time step were computed by considering a vehicle dynamics model, gear ratios and shift strategy during the cycle. The optimal solution for $\mu \in \{0, 5, 10, 20, 40, 80\}$ was obtained. The minimization in (7) was performed with the MATLAB Optimization Toolbox, using *constr*.

The total computation time on a Pentium II, 200 MHz PC was roughly 60 hours. This is unacceptable because the engineer needs to be able to evaluate many different parameter values for the LNT model, for example, and in addition, it was deemed important to include the TWC oxygen storage dynamics. Including a second state would result in approximately one month of computation time. Hence, to reduce the computation time, a new approximation is introduced.

4 Approximation via Local Engine Calibrations

The biggest time sink in the optimization process is the minimization operation performed by *constr*. The DISI model is very nonlinear, and results in many local minima. The idea of the following approximation is to replace the DISI engine model with a finite set of model behaviors, called *calibrations*, parameterized by engine speed and torque. More precisely, at each engine speed and torque point, the engine model is replaced by a finite set of possible feedgas characteristics, chosen in a way that they are likely to be useful in finding an approximate optimal policy. For the use of calibrations to develop "fixed structure" policies for complex DISI and hybrid diesel powertrains, see [9].

Quantize the engine speed and engine torque values by a finite grid:

$$\begin{array}{c} RPM \in \{\psi_1, \psi_2, \ldots, \psi_r\} \;\; (RPM) \\ Torque \in \{\varphi_1, \varphi_2, \ldots, \varphi_l\} \;\; (Nm) \end{array} \tag{8}$$

For each of the point (ψ_i, φ_j), a *normal calibration* is generated by minimizing the cost that represents the weighted sum of fuel consumption and NOx emissions

$$J = \text{fuel} + \lambda \cdot NOx, \tag{9}$$

for $\lambda \in \{0, 2, 5, 10, 30, 60, 80, 150\}$, over the engine parameters throttle position, fuel flow, EGR percent and spark. EGR percent is constrained to be between 0 and 30 for stratified, and 0 to 10 for homogeneous mode, and spark between 5 and 45 degrees (BTDC). The stratified and homogeneous regimes are treated separately during the optimization. Additional constraints are imposed that limit the intake manifold pressure between 5 and 100 kPa, torque equal to be φ_j, and engine speed equal to ψ_i, where $\varphi_j \in \{0, 6.25, 15, 25, 35, 45, 55, 65, 75, 85, 95, 105\}$, and $\psi_i \in \{600, 1250, 1750, 2250, 2750, 3250\}$.

For rich operation, the DISI model is used to generate a *purge calibration*. This is obtained by maximizing CO emissions entering the LNT. Since purge can only take place under rich conditions, the air-to-fuel ratio is constrained to be less than stoichiometry, and the combustion regime to be homogeneous.

Over the drive cycle, engine parameters are generated by interpolating calibrations of grided operating points (8) around the true operating point. Figure 3 com-

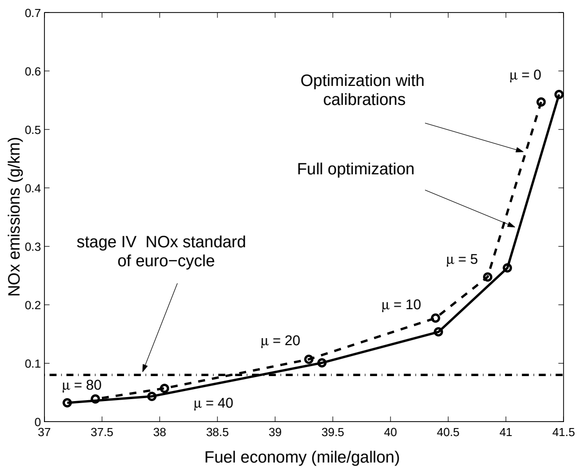

Figure 3: Fuel economy versus NOx emissions of optimal policy with calibration and from constrained optimization, over the Euro-cycle. The DISI engine and TWC models are quasi-static, and LNT model is dynamically updated.

pares the results of performing the dynamic programming with the engine calibrations versus the full optimization over the engine input parameters. This figure plots NOx emissions in g/km versus fuel economy in miles per gallon, over the Euro-cycle. It is seen that the results are very close. The time taken for generating the set of calibrations was roughly 4 hours (Pentium II, 200 MHz PC). However, once the calibration is done for the static DISI engine, the dynamic programming with different aftertreatment system parameters, or with different system configurations, can be easily and quickly done because a calibration can be repeatedly used due to its independence of the aftertreatment system.

5 Vectorization for Multi-State Models

The next step in developing dynamic programming as a realistic tool for tradeoff analysis was to consider models with more than one state. This would allow the consideration of important physical phenomena such as oxygen storage in the TWC and the temperature evolution of the aftertreatment elements. Using the method based on calibrations, and considering a one state model consisting of static TWC and the dynamic LNT NOx level studied in Sections 3 and 4, the discretized dynamic programming algorithm resulted in a computation time of 3 hours. It was determined that the major computation bottle neck during optimization was the interpolation operation (recall (7)). However, this can be remedied by interpolating on a vector scale. The basic idea is to build up the look-up tables for dynamics update of x, and instant cost $\bar{g}$, as a function of quantized state η_k, control input u, weight μ, and operating point (ψ_i, φ_j). Once these tables are loaded, they are 'vectorized' and used to update (7) on a vector scale during the dynamic programming. The time spent, based on calibrations generated in Section 4, is summarized in the Table 1.

Table 1: Time consumption on dynamic programming based on calibration. The Pentium II, 200 MHz PC was used for computation. To obtain total time consumption, time taken for calibration (4 hours) should be added.

aftertreatment system model	time taken (pointwise)	time taken (vectorized)
one state - static TWC - dynamic LNT	5 hours	20 minutes
two state - dynamic TWC - dynamic LNT	60 hours	40 minutes

6 Case Studies

This section considers several practical case studies that illustrate how design decisions can be made on the basis of optimization. The optimization is based on a static DISI model from [8], and a two state, dynamic model of the aftertreatment system. The dynamics of the TWC were limited to the oxygen storage phenomenon [5] since this is crucial for purging. The LNT model [4] is represented by the NOx storage level. Since many of the LNT model's parameters are quite temperature sensitive, a static model of LNT temperature was developed as a function of engine feedgas temperature. The state space is discretized as $x_{LNT} \times x_{TWC}$:

$$\begin{aligned} x_{LNT} &= \text{LNT\_max} \times \{0, 0.05, \ldots, 0.5, 0.7, 0.9, 1\} \\ x_{TWC} &= \text{TWC\_max} \times \{0, 0.25, 0.5, 0.75, 1\} \end{aligned} \tag{10}$$

where LNT_max and TWC_max represent maximum NOx trap capacity of LNT in grams and oxygen storage capacity of TWC in grams, respectively. The optimization is done with the interpolated DISI engine calibrations over the Euro-cycle.

6.1 Case Study 1: TWC and LNT Capacities

The capacity of the LNT to be used on a vehicle will be determined by a tradeoff between manufacturing price and system performance. To study this tradeoff, optimal solutions are obtained with various maximum trap capacities for the LNT:

$$\begin{aligned} &\text{LNT\_max} \in \{0.035, 0.075, 0.15, 0.3, 0.5\}, \\ &\text{TWC\_max} = 0.5 \end{aligned} \tag{11}$$

The fuel economy in miles per gallon, for the Stage IV NOx Emission Standard for the Euro-cycle (0.08 g/km), is shown in Figure 4 as a function of maximum trap capacity of LNT.

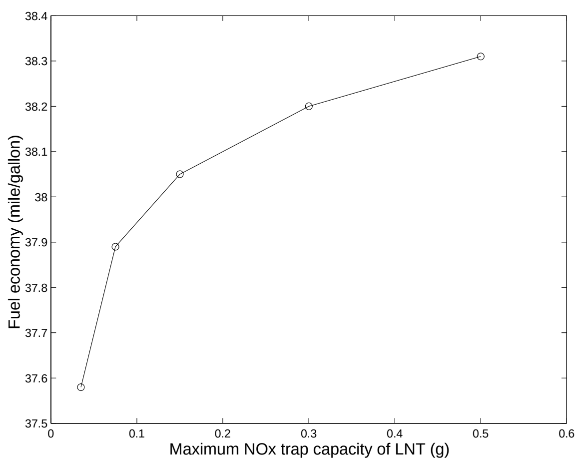

Figure 4: Fuel economy satisfying stage IV NOx emission standard of Euro-cycle with various maximum trap capacity of LNT.

It is seen that fuel economy improvement rapidly rolls off as trap capacity increases, and is mostly improved at low maximum trap capacity.

The effect of TWC oxygen storage capacity on fuel economy is also evaluated. The maximum oxygen storage capacity of TWC was varied over:

$$\begin{aligned} &\text{LNT\_max} = 0.15, \\ &\text{TWC\_max} \in \{0.125, 0.25, 0.5, 1, 2\} \end{aligned} \tag{12}$$

Figure 5 shows the fuel economy as a function of maximum capacity.

As can be seen, fuel economy decreases as maximum capacity of TWC increases. This is because purging is delayed until reductants, such as CO and H_2, are effectively delivered to the LNT, and the delay is proportional to the emptying time of the oxygen stored in the TWC.

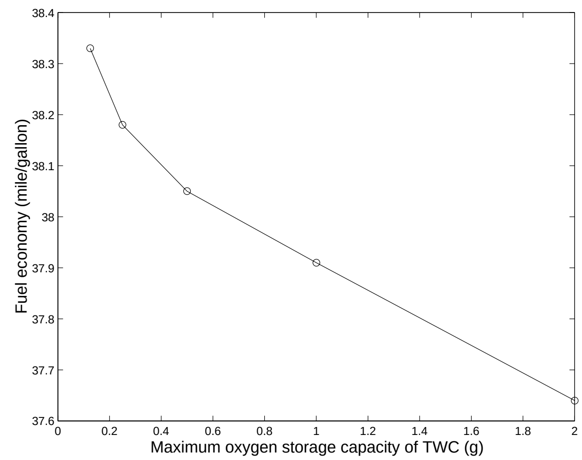

Figure 5: Fuel economy satisfying stage IV NOx emission standard of Euro-cycle with various maximum oxygen storage capacity of TWC.

6.2 Case Study 2: Removal of Homogeneous Lean Mode

The homogeneous lean mode is limited to air-to-fuel ratios from 15 to 20. In the study, the removal of homogeneous lean mode is considered in order to simplify the engine operation and control strategy. The effect of removal is evaluated by dynamic programming, and the fuel economy and NOx emissions over the Euro-cycle are shown in Figure 6.

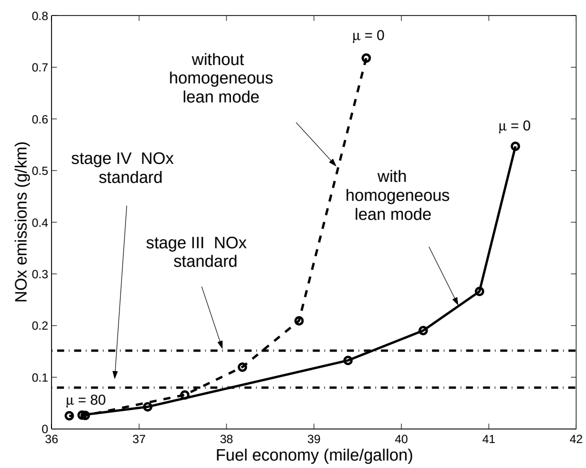

Figure 6: Fuel economy and NOx emissions over Euro-cycle with, and without homogeneous lean mode.

The figure shows that the loss of fuel economy without the homogeneous lean mode is 0.3 miles per gallon, which corresponds to a 0.92 % loss, with Stage IV NOx Emission Standard of Euro-cycle. However, for Stage III NOx Emission Standard (0.15 g/km), the loss of fuel economy is 1.4 miles per gallon. This is a 3.51 % loss, which is not acceptable.

7 Conclusions

In this paper, we treated a problem of predicting the best emission constrained fuel economy of a direct injection spark ignition powertrain over a drive cycle. This problem is difficult because the search of the optimal trajectory has to be done over all possible trajectories of the engine and the aftertreatment on a drive cycle. The search procedure is based on the dynamic programming (DP) algorithm. The procedure is made computationally tractable by combining several ideas that involve (i) model simplification; (ii) state and control discretization; (iii) restricting the search to a smaller set of trajectories that, based on engineering judgment, are deemed likely to contain the optimal policy, and (iv) careful treatment of computer implementation details. Numerical results have demonstrated significant reduction in the computation time, while near optimal solutions are generated.

The procedure has been used in several case studies where the effect of adjusting hardware parameters or control strategy on the fuel economy was evaluated. The ability to conduct assessments of this kind is very important early on during the development of an automotive system and its control strategy.

This study resulted from a cooperative research project between researchers from Ford Research Laboratory and researchers from the University of Michigan. It demonstrates how advanced optimization techniques can be adapted to a realistic industrial problem.

Acknowledgments

The authors thank Jeff Cook, Jing Sun, Michiel van Nieuwstadt and Yanying Wang of Ford Research Laboratory for helpful discussions. The work of J.M. Kang and J.W. Grizzle is supported by NSF GOALI grant, ECS-9631237, with matching funds from Ford Motor Company.

References

[1] M. S. Brogan, R. J. Brisley, A. P. Walker, D. E. Webster, W. Boegner, N. P. Fekete, M. Krämer, B. Krutzsch and D. Voigtländer, "Evaluation of NO_x Storage Catalyst as an Effective System for NO_x Removal from the Exhaust Gas of Leanburn Gasoline Engines," *SAE Paper 952490.*

[2] N. Fekete, R. Kemmler, D. Voigtländer, B. Krutzsch, E. Zimmer, G. Wenninger, W. Strehlau, J. A. A. van den Tillaart, J. Leyrer, E. S. Lox and W. Müller, "Evaluation of NO_x Storage Catalyst for Lean Burn Gasoline Fueled Passenger Cars," *SAE Paper 970746.*

[3] D. P. Bertsekas, *Dynamic Programming and Optimal Control*, Athena Scientific, 1995.

[4] Y. Wang, S. Raman and J. W. Grizzle, "Lean NO_x Trap Modeling for Lean Burn Engine Control," *Proc. of 1999 American Control Conference,* San Diego, Ca, 1999.

[5] E. P. Brandt, *Modeling and Diagnostics of Three-way Catalysts for Advanced Emissions Control Systems*, Ph.D. Thesis, University of Michigan, 1998.

[6] E. P. Brandt, Y. Wang and J. W. Grizzle, "A simplified three-way catalyst model for use in on-board SI engine control and diagnostics," *Proc. of ASME Dynamic Systems and Control Division*, Vol. 61, pp. 653-659, 1997.

[7] J. B. Heywood, *Internal Combustion Engine*, McGraw-Hill, 1988.

[8] J. Sun, I. Kolmanovsky, D. Brehob, J. A. Cook, J. Buckland and M. Haghgooie, "Modeling and Control of Gasoline Direct Injection Stratified Charge (DISI) Engines," *Proc. of 1999 IEEE Conference on Control Applications*, Hawaii.

[9] I. Kolmanovsky, M. van Nieuwstadt and J. Sun, "Optimization of Complex Powertrain systems for Fuel Economy and Emissions," *Proc. of 1999 IEEE Conference on Control Applications,* Hawaii, 1999.

[10] A. P. de Madrid, S. Dormido, F. Morilla, "Reduction of the Dimensionality of Dynamic Programming: A Case Study," *Proc. of 1999 American Control Conference,* San Diego, Ca, 1999.