SYNTHESIS OF NONLINEAR OBSERVERS VIA STRUCTURAL ANALYSIS AND NUMERICAL DIFFERENTIATION

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Abstract

Recent work in nonlinear observer design has sought to bring together the structural (or transformation based) approach found in [8, 5, 1, 11] and the numerical differentiation approach found in [4]. The goal is to achieve higher flexibility and wider applicability than is possible with either approach alone. In this paper, it is noted that any observable, autonomous system admitting an equilibrium point can be expressed as a state-affine system parameterized by the output and its derivatives. A class of (single-output) systems is identified that admit parameterizations with *fewer* than $n-1$ derivatives, where n is the dimension of the state.

1 Introduction and Background

The goal of the structural approach to observer design is to transform a given nonlinear system

$$\begin{array}{rcl} \dot{x} & = & f(x,u) \\ y & = & h(x) \end{array} \tag{1}$$

into an observable linear system plus nonlinear input-output injection. In this way, an observer with linear, exponentially converging, error dynamics can be readily formed. The first results in [8, 17] targeted single-output systems that can be transformed, via a (local) state space diffeomorphism, into

$$\begin{array}{rcl} \dot{x} & = & Ax + \phi(u,y) \\ y & = & Cx. \end{array} \tag{2}$$

Necessary and sufficient conditions for the existence of the transformation are known [8]. It is also known that the class of systems for which observers can be constructed in this way is very small.

Subsequent research has addressed the case of multi-output systems, but even more importantly, it has sought to expand the class of systems that can be treated in two directions: the first by allowing output transformations [9, 19]; and the second by allowing a more general class of target systems [1, 2, 5, 10, 11, 12, 14, 15, 20]. Among the latter group, it is worth recalling the work of [14], which allowed more general output injections of the form

$$\begin{array}{rcl} \dot{x} & = & Ax + \phi(y, \dot{y}, \ldots, y^{(r)}, u, \dot{u}, \ldots, u^{(s)}) \\ y & = & Cx, \end{array} \tag{3}$$

while keeping the matrices A and C constant, and the work of [1, 10, 11], which allowed the "A" matrix to be time varying in the sense that it depends on the input and output

$$\begin{array}{rcl} \dot{x} & = & A(\nu)x + \phi(\nu) \\ y & = & Cx, \end{array} \tag{4}$$

where $\nu = (u, y)$.

The motivation for seeking systems in the "state-affine" form (4) comes from [3, 6], which established conditions under which linear time-varying observer design could be extended to this class of systems. The observer is

$$\begin{array}{rcl} \dot{\hat{x}} & = & A(\nu)\hat{x} + \phi(\nu) + L(t)(y - \hat{y}) \\ \hat{y} & = & C\hat{x} \\ \dot{S} & = & -\theta S - A^T(\nu)S - SA(\nu) + C^TC, \theta > 0 \\ L(t) & = & S^{-1}(t)C^T. \end{array} \tag{5}$$

The conditions given for the error dynamics to converge exponentially to zero are that solutions to (4) exist forward in time on $[t_0, \infty)$, and that there exist finite numbers $T > 0$ and $\alpha > 0$ such that the standard time-varying observability grammian is greater than αI over every interval of the form [t, t+T], for $t \geq t_0$. These conditions are not readily checkable for most systems.

There are two main shortcomings to the structural approach to observer design. The first is that despite much recent progress, the class of systems that can be treated remains quite restricted. The second and more important shortcoming is that computing the appropriate coordinate transformation can be very difficult. In many cases, it requires the solution of a partial differential equation (PDE), which is enough to deter many would-be users.

A completely different approach to obtaining state estimates of a system (1) was outlined in [4]. The idea is based directly on the notion of observability, namely, the ability to reconstruct the states on the basis of the system's output and a finite number of its derivatives. Numerical differentiation was used to compute the derivatives of the output, and the states were then reconstructed from them with a static map. The approach may seem somewhat naive, but it is very general, flexible and straightforward to implement in practice: it does not require the solution of a PDE to find an appropriate change of coordinates. Moreover, it is quite possible to state "separation-type" results for such observers, along the line of the observation made in [13] for discrete-time systems. Finally, for systems expressed in input-output form, namely,

$$y^{(n)} = P(y, \dot{y}, \ldots, y^{(n-1)}, u),$$

or equivalently,

$$\begin{array}{rcl} \dot{x}_i &=& x_{i+1} \quad i = 1, \ldots, n-1 \\ \dot{x}_n &=& P(x_1, x_2, \ldots, x_n, u) \\ y &=& x_1, \end{array} \tag{6}$$

observers are computing the output and its derivatives, in any case, and this fact is explicitly exploited in the high gain observer found in [7, 18], for example, and the separation theorem of [16].

Among the shortcomings of this approach, one must note that the numerical derivatives are quite sensitive to measurement noise if the derivatives must be computed within a short delay of taking the measurements [4], that is, if an interval of time for smoothing cannot be allowed. Thus, it is best to minimize, as much as possible, the maximum order of the derivatives that must be computed.

Our goal here is to contribute to the merger of the structural and numerical approaches in order to combine the strengths of each approach and to ameliorate the shortcomings. Already, from the work of [14], this is seen to be a natural thing to do since the output injection terms require the computation of the derivatives of the output from measured data.

2 An Observation on State-Affine Parameterizations

To simplify the exposition, systems without inputs will be treated in the remainder of the paper. Consider a system of the form (1), with $x \in I\!R^n$, no inputs, $y \in I\!R$, and $f(x, u) =: f(x)$ analytic. Suppose that the following observability property (see [4]) is met: there is an integer N such that the map defined by

$$H(x) = \left(y, \dot{y}, \cdots, y^{(N-1)}\right)', \tag{7}$$

is *injective*. In a neighborhood of a point x_0, condition (7) is implied by the *observability rank condition*

$$\dim\{dh(x_0), \cdots, dL_f^{(n-1)}h(x_0)\} = n. \tag{8}$$

Since $y \in I\!R$, $N \geq n$. By the injectivity of H, there exists L such that $x = L\left(y, \dot{y}, \cdots, y^{(N-1)}\right)$.

Suppose in addition that $f(0) = 0$. Then the mean value theorem can be used to show that there exists a (non-unique) matrix $A(x)$ such that

$$f(x) = A(x)x. \tag{9}$$

Supposing that (1) is observable, the system can be rewritten as

$$\begin{array}{rcl} \dot{x} &=& A \circ L(y, \dot{y}, \cdots, y^{(N-1)})x \\ &=:& \bar{A}(y, \dot{y}, \cdots, y^{(N-1)})x \\ y &=& x_1, \end{array} \tag{10}$$

which is of the form (4), with $\nu = (y, \dot{y}, \cdots, y^{(N-1)})$. Thus, under linear time-varying-like, uniform observability conditions, an observer of the form (5) can be designed, as long as the derivatives of the output can be obtained. These can be computed via the numerical differentiation results of [4], for example. Designing an observer on the basis of (10) bears some resemblance to an extended Kalman filter; the difference is that the system is being "linearized" along a state "estimate" coming from numerical differentiation instead of an estimate coming from the filter itself. It should be noted however that meeting the observability requirement can be difficult, and seems to depend on the choice of A and the coordinates in which the parameterization is done.

Of course, even without an equilibrium point assumption, (1) could be expressed as

$$\begin{array}{rcl} \dot{x} &=& f \circ L(y, \dot{y}, \cdots, y^{(N-1)}) \\ &=:& \bar{\phi}(y, \dot{y}, \cdots, y^{(N-1)}) \\ y &=& x_1. \end{array} \tag{11}$$

However, the observability condition of (5) would then be impossible to meet. More generally speaking, it is the experience of the authors, that, when trying to apply observers of the form (5) to systems with noisy (or inexact) measurements, it is "best" to include as much of the model as possible in the "$A(\nu)$" matrix, and to minimize the part that is placed in the injection term. Here, "best" is meant in the intuitive sense of achieving some "filtering" or "smoothing" of the measurement perturbation. One way of looking at a filter is that it is a means of producing trajectories that are compatible with a given model. Terms in the "$A(\nu)$" matrix seem to force more constraints on the trajectories than those in the injection because the observer treats anything in the injection term as an exogenous variable. On the other hand, this must be balanced against the observability of the resulting state-affine system, which is dependent on how the terms are divided up between A and ϕ.

As a final point of "philosophy", it is clear that the higher the order of the derivative, the less exact will be its estimate. Thus, it is important to use as few derivatives as possible when expressing a system in state-affine form. On the other hand, placing constraints on the maximum order of the derivatives allowed in (10) restricts the class of systems that will meet the associated conditions.

3 Main Results

As mentioned in Section 1, previous work has studied the problem of transforming a system to a state-affine form without considering the use of derivatives of the output in A, while allowing them in ϕ. In fact, there seems to be very little reason to allow derivatives in one and not the other. One of the contributions of this paper is to show that it is possible to identify a *subclass* of systems for which constructive necessary and sufficient conditions can be given for the existence of a state coordinate transformation putting the system into state-affine form with at most $n-2$ output derivatives. The problem is looked at from both the state space and input-output perspectives.

3.1 State-space perspective

Lemma 1 *In a neighborhood of a given point x_0, there exists a local change of coordinates $z = \psi(x)$ which transforms $\dot{x} = f(x)$ into*

$$\dot{z} = A(z_1, \cdots, z_{n-1}) \cdot z + \phi(z_1, \cdots, z_{n-1}), \qquad (12)$$

if, and only if, there exists a vector field X satisfying $X(x_0) \neq 0$ and $[X, [X, f]] = 0$.

Proof. If such a vector field exists, then choose coordinates $(z_1, \cdots, z_n)$ in which $X = \frac{\partial}{\partial z_n}$. The condition $[X, [X, f]] = 0$ implies that $\frac{\partial^2 f}{\partial z_n^2} = 0$, which gives (12). On the other hand, it is easily verified that if the system has the form (12), then $X = \frac{\partial}{\partial z_n}$ satisfies the stated conditions. ∎

The following result gives a sufficient condition under which the system can be taken into the state-affine form (4), with $\nu = (y, \dot{y}, \cdots, y^{(n-2)})$.

THEOREM 1 *Consider the scalar output system*

$$\begin{aligned} \dot{x} &= f(x) \\ y &= h(x), \end{aligned} \qquad (13)$$

with $x \in I\!R^n$, $n \geq 3$. In a neighborhood of a given point x_0, suppose that the observability rank condition (8) holds and that there exists a vector field X satisfying $X(x_0) \neq 0$, $X \in \{dh, \cdots, dL_f^{n-2}h\}^{\perp}$ and $[X, [X, f]] = 0$. Then there exists a local change of coordinates $z = \psi(x)$ in which the system can be expressed as

$$\begin{aligned} \dot{z}_1 &= z_2 \\ &\vdots \\ \dot{z}_{n-2} &= z_{n-1} \\ \dot{z}_{n-1} &= A_{n-1}(z_1, \cdots, z_{n-1}) z_n \\ \dot{z}_n &= A_n(z_1, \cdots, z_{n-1}) z_n + \phi_n(z_1, \cdots, z_{n-1}) \\ y &= z_1 \end{aligned} \qquad (14)$$

Proof. From $\dim\{dh(x_0), \cdots, dL_f^{(n-2)}h(x_0)\} = n-1$, and the existence of a vector field X satisfying $X(x_0) \neq 0$ and $X \in \{dh, \cdots, dL_f^{n-2}h\}^{\perp}$, it can be shown that, in a neighborhood of x_0, there exists a function $\eta(x)$ such that $(z_1, \ldots, z_{n-1}, \tilde{z}_n) := (h(x), \ldots, L_f^{(n-2)}h(x), \eta(x))$ is a local change of coordinates, and, in these coordinates, $X = \frac{\partial}{\partial \tilde{z}_n}$. From this, it easily follows that $\dot{z}_i = z_{i+1}$ for $i = 1, \cdots, n-2$. As in the proof of Lemma 1, the condition $[X, [X, f]] = 0$ gives the affine structure in the remaining coordinates:

$$\begin{aligned} \dot{z}_{n-1} &= A_{n-1}(z_1, \cdots, z_{n-1}) \tilde{z}_n + \tilde{\phi}_{n-1}(z_1, \cdots, z_{n-1}) \\ \dot{\tilde{z}}_n &= \tilde{A}_n(z_1, \cdots, z_{n-1}) \tilde{z}_n + \tilde{\phi}_n(z_1, \cdots, z_{n-1}). \end{aligned}$$

From the observability rank condition, it follows that A_{n-1} does not vanish in a neighborhood of x_0. Then, the change of coordinates $z_n := \tilde{z}_n + \frac{\tilde{\phi}_{n-1}}{A_{n-1}}$ puts the system into the desired form. ∎

Of course, (14) can be re-written as

$$\begin{aligned} \dot{z} &= A(\nu) \cdot z + \phi(\nu) \\ y &= C \cdot z \end{aligned} \qquad (15)$$

with $\nu = (y, \dot{y}, \cdots, y^{(n-2)})$,

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \cdots & 1 & 0 \\ 0 & 0 & \cdots & \cdots & 0 & A_{n-1}(\nu) \\ 0 & 0 & \cdots & \cdots & 0 & A_n(\nu) \end{pmatrix} \qquad (16)$$

$$\phi = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \phi_n(\nu) \end{pmatrix} \qquad (17)$$

and

$$C = (1, 0, \cdots, 0). \qquad (18)$$

It is remarked that Theorem 1 can be used to show that an observable system of the form

$$\dot{z}_1 = A_1(z_1) z_2 + \phi_1(z_1)$$

$$\begin{array}{rcl} & \vdots & \\ \dot{z}_{n-2} & = & A_{n-2}(z_1,\cdots,z_{n-2})z_{n-1}+\phi_{n-2}(z_1,\cdots,z_{n-2}) \\ \dot{z}_{n-1} & = & A_{n-1}(z_1,\cdots,z_{n-1})z_n+\phi_{n-1}(z_1,\cdots,z_{n-1}) \\ \dot{z}_n & = & A_n(z_1,\cdots,z_{n-1})z_n+\phi_n(z_1,\cdots,z_{n-1}) \\ y & = & z_1 \end{array} \quad (19)$$

can always be expressed in the form (14) through a change of coordinates.

3.2 Input-output perspective

This section gives a *constructive* necessary and sufficient condition for the existence of a state coordinate transformation which takes system (13) into the state-affine form (15). The procedure is first summarized as a sequence of steps, and then formalized as a theorem.

ALGORITHM: Suppose that (13) satisfies the observability rank condition (8), in a neighborhood of a point x_0 so that it has a local input-output representation

$$y^{(n)} \quad = \quad P(y,\dot{y},\ddot{y},\ldots,y^{(n-1)}). \qquad (20)$$

Let $(y_0,\cdots,y_0^{(n-1)}) := (h(x_0),\ldots,\mathrm{L}_f^{(n-1)}h(x_0))$.

Step 1 If $\mathrm{d}[\frac{\partial^2 P}{\partial {y^{(n-1)}}^2}]\wedge \mathrm{d}y\wedge\mathrm{d}\dot{y}\wedge\cdots\wedge\mathrm{d}y^{(n-2)}=0$, then there exists a function $A_{n-1}(y,\dot{y},\cdots,y^{(n-2)})$ solving

$$\begin{array}{rcl} \frac{\partial A_{n-1}}{\partial y^{(n-2)}}-\frac{1}{2}\cdot\frac{\partial^2 P}{\partial {y^{(n-1)}}^2}\cdot A_{n-1} & = & 0 \\ A_{n-1}(y_0,\cdots,y_0^{(n-1)}) & \neq & 0 \end{array} \qquad (21)$$

■

Step 2 Define the polynomial $\tilde{P}(y,\dot{y},\ddot{y},\cdots,y^{(n-1)})$ by

$$\tilde{P} \quad = \quad P-\frac{\dot{A}_{n-1}}{A_{n-1}}\cdot y^{(n-1)}, \qquad (22)$$

where $\dot{A}_{n-1} := \sum_{i=0}^{n-2}\frac{\partial A_{n-1}}{\partial y^{(i)}}\cdot y^{(i+1)}$. If $\mathrm{d}[\frac{\partial\tilde{P}}{\partial y^{(n-1)}}]\wedge\mathrm{d}y\wedge\mathrm{d}\dot{y}\wedge\cdots\wedge\mathrm{d}y^{(n-2)}=0$, then a function $A_n(y,\dot{y},\cdots,y^{(n-2)})$ is defined by $A_n=\frac{\partial\tilde{P}}{\partial y^{(n-1)}}$.

■

Step 3 Let the polynomial $\bar{P}(y,\dot{y},\ddot{y},\cdots,y^{(n-2)})$ denote

$$\bar{P} \quad = \quad \frac{\tilde{P}-A_n\cdot y^{(n-1)})}{A_{n-1}}. \qquad (23)$$

If $\mathrm{d}[\bar{P}]\wedge\mathrm{d}y\wedge\mathrm{d}\dot{y}\wedge\cdots\wedge\mathrm{d}y^{(n-2)}=0$, then a function $\phi_n(y,\dot{y},\cdots,y^{(n-2)})$ is defined by $\phi_n=\bar{P}$.

■

A constructive necessary and sufficient condition for the existence of a state coordinate transformation which takes (13) into (15) is given next.

THEOREM 2 *In a neighborhood of a point x_0 where the observability rank condition holds, (13) is equivalent to (15) under a state coordinate transformation if, and only if,*

$$\begin{array}{rcl} d[\frac{\partial^2 P}{\partial {y^{(n-1)}}^2}]\wedge dy\wedge d\dot{y}\wedge\cdots\wedge dy^{(n-2)} & = & 0 \\ d[\frac{\partial\tilde{P}}{\partial y^{(n-1)}}]\wedge dy\wedge d\dot{y}\wedge\cdots\wedge dy^{(n-2)} & = & 0 \\ d[\bar{P}]\wedge dy\wedge d\dot{y}\wedge\cdots\wedge dy^{(n-2)} & = & 0 \end{array} \qquad (24)$$

with the polynomials P, $\tilde{P}$ and $\bar{P}$ defined by (20), (22) and (23), respectively.

Proof of necessity. Suppose that there exists a coordinate transformation which takes (13) into (15). The polynomial P, which is equal to $y^{(n)}$, then can be written as

$$\begin{array}{rcl} P & = & \dot{A}_{n-1}\cdot z_n+A_{n-1}\cdot\dot{z}_n \\ & = & \frac{\dot{A}_{n-1}}{A_{n-1}}\cdot y^{(n-1)}+A_{n-1}\cdot(A_n\cdot z_n+\phi_n) \\ & = & \frac{\dot{A}_{n-1}}{A_{n-1}}\cdot y^{(n-1)}+A_{n-1}\cdot(A_n\cdot\frac{y^{(n-1)}}{A_{n-1}}+\phi_n) \\ & = & \frac{1}{A_{n-1}}\cdot[\sum_{i=0}^{n-2}\frac{\partial A_{n-1}}{\partial y^{(i)}}\cdot y^{(i+1)}]\cdot y^{(n-1)}+A_n\cdot y^{(n-1)} \\ & & +A_{n-1}\cdot\phi_n. \end{array}$$

Step 1 Applying the algorithm, yields

$$\frac{\partial^2 P}{\partial {y^{(n-1)}}^2} \quad = \quad \frac{2}{A_{n-1}}\cdot\frac{\partial A_{n-1}}{\partial y^{(n-2)}}.$$

The first condition of (24) is thus satisfied. The function A_{n-1} depends on $(y,\dot{y},\cdots,y^{(n-2)})$ and is a solution of (21).

Step 2 The polynomial $\tilde{P}$ is determined from (22)

$$\begin{array}{rcl} \tilde{P} & = & P-\frac{\dot{A}_{n-1}}{A_{n-1}}\cdot y^{(n-1)} \\ & = & A_n\cdot y^{(n-1)}+A_{n-1}\cdot\phi_n. \end{array}$$

This yields $\frac{\partial\tilde{P}}{\partial y^{(n-1)}} = A_n$. The second condition of (24) is thus satisfied; the function A_n depends on $(y,\dot{y},\cdots,y^{(n-2)})$.

Step 3 The polynomial $\bar{P}$ is determined from (23)

$$\begin{aligned}\bar{P} &= \frac{\tilde{P} - A_n \cdot y^{(n-1)}}{A_{n-1}} \\ &= \phi_n\end{aligned}$$

The third condition of (24) is thus satisfied; the function ϕ_n depends on $(y, \dot{y}, \cdots, y^{(n-2)})$.

The conditions of Theorem 2 are thus necessary for the existence of a state representation of the given form. ∎

Proof of sufficiency. If the conditions of Theorem 2 are satisfied, then functions A_{n-1}, A_n and ϕ_n can be derived from the algorithm. It is then straightforward to show $(x_1, \ldots, x_{n-1}, x_n) := (y, \ldots, y^{(n-2)}, \frac{y^{(n-1)}}{A_{n-1}})$ yields a realization of the form (15). ∎

4 Example

Consider the system

$$\begin{aligned}\dot{x}_1 &= x_2 + \tfrac{1}{2}D_3(x) \\ \dot{x}_2 &= (1 + \tfrac{1}{2}x_2^2)x_3 \\ \dot{x}_3 &= D_3(x) \\ y &= x_1 - x_2,\end{aligned} \tag{25}$$

where,

$$\begin{aligned}D_3(x) &= 20[1 - 2(x_1 - x_2)^2 - 2(x_2 - x_3)^2](x_2 - x_3) \\ &\quad -20(x_1 - x_2) - 10x_3.\end{aligned}$$

Using Theorem 2, one verifies that system (25) is equivalent to (15) with $A_2 = (1 + 0.5(\dot{y})^2)$, $A_3 = -10$ and $\phi = (0, 0, -20y + 20(1 - 2(y^2 + \dot{y}^2))\dot{y})'$. The coordinate transformation is $z_1 = x_1 - \frac{1}{2}x_3$, $z_2 = x_2$ and $z_3 = x_3$.

We next note that, in this case, the injection term can be re-absorbed into the A-matrix and write the system in the form (15) with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & (1 + 0.5(\dot{y})^2) \\ -20 & 20(1 - 2(y^2 + \dot{y}^2)) & -10 \end{bmatrix} \tag{26}$$

and

$$\phi = [0, 0, 0]'. \tag{27}$$

Three observers were computed for the system: one using the numerical differentiation approach (computing $y, \dot{y}, \ddot{y}$), a second observer based upon (10) and the third using the observer (5), based on (26) and (27). The actual measurement used for the estimation process was assumed[1] to be $y_m = y + \text{noise}$. For the estimation of the numerical derivatives, the interpolating polynomial was of order 3, the sampling interval was chosen as 0.05, the moving window was of length 10, and the derivatives were estimated at the eighth node. The consequence of the latter choice is that the estimates are made with a delay of 0.1 seconds. The relationship between the states of (15) and the outputs is given by $z_1 = y$, $z_2 = \dot{y}$ and $z_3 = y^{(2)}/(1 + 0.5\dot{y}^2)$.

The results of the three observers are displayed in Figures 1, 2 and 3, respectively. The figures compare the true system states delayed by 0.1 seconds (solid line) and the estimated states (dashed line).

[1]Simulink block, `band-limited white noise`, with noise power equals $2e-6$ and sample time set to 0.15 seconds.

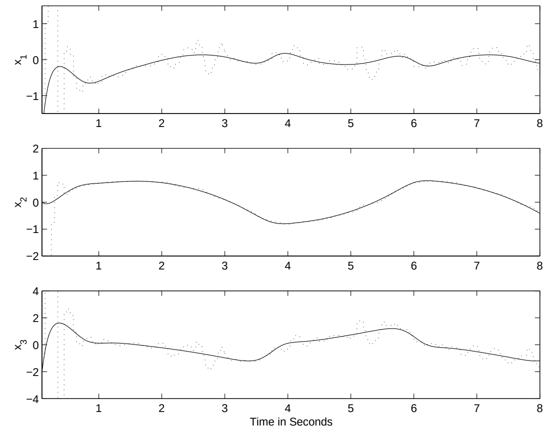

Figure 1: Comparison of the true system states delayed by 0.1 seconds (solid line) and the estimated states (dashed line) as determined by numerical differentiation.

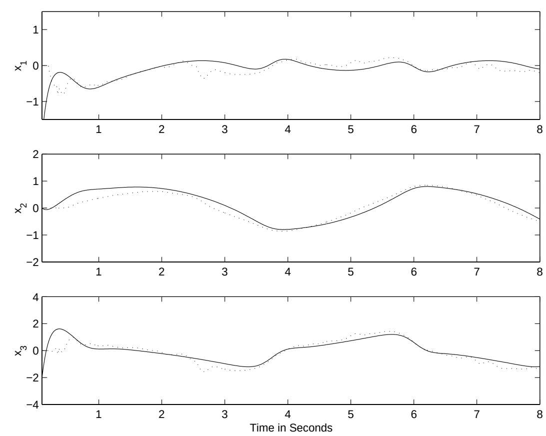

Figure 2: Comparison of the true system states delayed by 0.1 seconds (solid line) and the estimated states (dashed line) as determined by the state affine observer parametrized by y, $\dot{y}$ and $\ddot{y}$.

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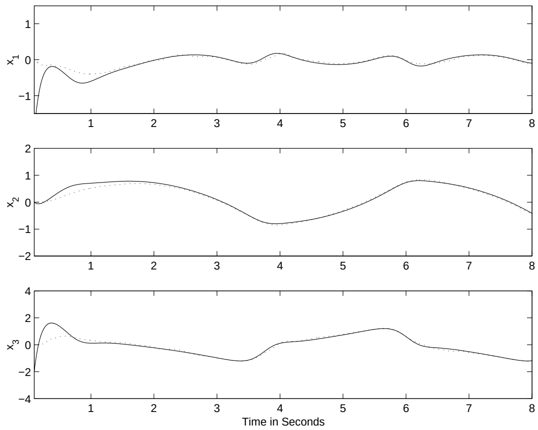

Figure 3: Comparison of the true system states delayed by 0.1 seconds (solid line) and the estimated states (dashed line) as determined by the state affine observer parametrized by y and $\dot{y}$.

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