Dynamic Optimization of Lean Burn Engine Aftertreatment

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Abstract

The competition to deliver fuel efficient and environmentally friendly vehicles is driving the automotive industry to consider ever more complex powertrain systems. Adequate performance of these new highly interactive systems can no longer be obtained through traditional approaches, which are intensive in hardware use and final control software calibration. This paper explores the use of Dynamic Programming to make model-based design decisions for a lean burn, direct injection spark ignition engine, in combination with a three way catalyst and an additional three-way catalyst, often referred to as a lean NOx trap. The primary contribution is the development of a very rapid method to evaluate the tradeoffs in fuel economy and emissions for this novel powertrain system, as a function of design parameters and controller structure, over a standard emission test cycle.

1 Introduction

Designing a powertrain system to meet drivability, fuel economy and emissions performance requirements is a complicated task. There are many tradeoffs to be analyzed in terms of which components to use, such as lean burn technology versus classical components, characteristics of individual components, such as size or temperature operating range, and the control policies to be employed. In addition, there are tradeoffs to be analyzed among the performance metrics themselves, such as emissions versus fuel economy. In the past, most of the powertrain design decisions were on the basis of hardware, that is, on the basis of assembling and evaluating many possible system configurations. Today, the time-line for vehicle design is constantly shrinking, the number of possible powertrain configurations is expanding, and the cost of doing hardware evaluations is growing. It is simply no longer feasible, economically, or time-wise, to make all (or even most) of the design decisions on the basis of hardware alone. More and more of the decisions must be made upon the basis of mathematical models and analysis.

This paper will describe the use of Dynamic Programming to assist in making powertrain design decisions on the basis of component models. The specific technology configuration analyzed here involves a direct injection spark ignition (DISI) engine. In this type of engine, fuel is injected directly into the combustion chamber during the compression stroke, and the highly concentrated fuel around the spark plug and extensive air motion enables combustion of an overall lean mixture (the shape of the piston is specially designed to enhance air motion (swirl or tumble), and it is further enhanced in the compression process) [1]. The DISI engine studied here can operate in either homogeneous or stratified mode. In stratified mode, the engine can operate at air-fuel ratios up to 40:1. The current NOx removal technique is to place an additional TWC, referred to as a lean NO_x trap (LNT), after the existing TWC in the exhaust system. NO_x is trapped in the LNT while the engine operates at a lean condition. By periodically operating the engine at a rich condition (in homogeneous mode), the trapped NO_x is purged and converted to N_2 by reductants such as CO, HC and H_2 [2, 3, 4]. The duration and frequency of the purging mode (rich operation of the engine), and obviously the control strategy for purging the LNT should be well optimized to achieve high fuel economy and low NO_x emissions.

Section 2 and Section 3 briefly discuss the models used for optimization, and set up the fuel economy versus emissions tradeoff problem in the context of a Dynamic Programming problem, respectively. Section 4.1 explores the results of the application of the standard state space discretization methods; it will be seen that the computation times are too long for the engineer to do case study analysis. Section 4.2 introduces a method for rapidly generating approximate solutions; a simple case is analyzed to show that the method can potentially produce near optimal solutions. The computation time is reduced by a factor of twenty. Section 4.3 points out how the computation speed can be further enhanced through the vectorization of the MATLAB code. Section 5 looks at several case studies using this optimization tool.

2 Models

It is well-known that the computation time of the Dynamic Programming algorithm is exponential in the number of states. For this reason, it is important to make a judicious choice of the complexity of the dynamic models used in the optimization. The LNT is a dynamic device in the sense that its capability to trap oxidants (NO_x and oxygen) changes dynamically until it reaches saturation, and similarly, the TWC dynamically stores and releases oxidants in the feedgas. The NO_x fill time of the LNT is on the order of 30 seconds to 1 minute, and its purge time is on the order of a few seconds. Finally, the most important dynamics of the engine are the intake manifold filling/emptying, which have a time constant on the order of a 100 milli-seconds. It is concluded from this that the dominant dynamics are those of TWC oxygen storage, LNT NO_x filling and emptying. Consequently, the engine can be treated as a static device delivering torque and exhaust feedgas (emissions concentrations, flow rates, temperature) as a function of throttle position, fuel flow, spark and exhaust gas recirculation (EGR) rate.

Control-oriented dynamic models of the TWC and LNT have been developed in [5, 6] and [4], respectively, and a mean-value model of a 1.8 L, 4 cylinder DISI engine has been developed in [7].

3 Mathematical Problem Formulation

A finite horizon optimization problem for determining a control strategy of the combined DISI engine and exhaust aftertreatment system depicted in Fig. 1 is posed in this section.

A model of the combined engine and emissions systems, discretized for numerical optimization, can be expressed as:

$$x(k+1) = f(x(k), u(k), \omega(k)) \tag{1}$$

$$y(k) = h(x(k), u(k), \omega(k)), \tag{2}$$

where $u(k)$ is the vector of engine input parameters such as throttle position, fuel mass flow rate, spark timing and EGR rate, $x(k)$ is the vector of states of the overall system, $\omega(k)$ is the vector of engine speed and load imposed by drive cycle as explained in Appendix, and $y(k)$ is the tailpipe NO_x emissions out of the LNT.

The objective of the study is to evaluate the tradeoff in fuel economy and NO_x emissions[1]. The instantaneous cost is chosen as a weighted sum:

$$g(y(k), u(k), \omega(k)) = \text{fuel}(k) + \mu \cdot NO_{x,tp}(k) = \text{fuel}(k) + \mu \cdot y(k). \tag{3}$$

In general, the emission performance of a vehicle is evaluated through a specific drive test cycle such as the US FTP cycle, or the European Drive Cycle. Then the objective is to find the optimal control input, $u(k)$, that minimizes the cost functional

$$J(x) = \min_{u \in U} \sum_{k=0}^{M-1} g(y(k), u(k), \omega(k)) = \min_{u \in U} \sum_{k=0}^{M-1} \bar{g}(x(k), u(k), \omega(k)), \tag{4}$$

[1]Since a DISI engine is mostly operated in a lean mode, it is felt that CO and HC levels should not be problem. The only exception would occur if the LNT is purged too often, which would also show up as a fuel economy penalty.

where U represent constraints for u imposed by meeting the load demands of the specific drive cycle, plus constraints like intake manifold pressure being positive and not exceeding one atmosphere (unless boosted); M is the time length of the drive cycle.

Remark: The constraint to meet the load requirements of the Euro-cycle imposes a relationship on the inputs, u=(throttle, fuel, spark, EGR). This is taken into account in the formulation.

The cost (4) represents the cumulative weighted sum of fuel consumption and tailpipe NO_x emissions over the drive cycle. The objective will be to minimize the cost function (4), for a range of μ. This will provide information on the sensitivity of fuel economy to tailpipe NO_x emission levels, and is more useful than just knowing the best fuel economy for a given emissions constraint. A systematic solution to the above problem can be determined recursively via Bellman's Dynamic Programming [8] as follows:

Step $M-1$:

$$\begin{aligned} J_{M-1}(x(M-1)) \quad &:= \min\nolimits_{u(M-1)\in U(M-1)} \left[g(y(M-1), u(M-1), \omega(M-1))\right] \\ &= \min\nolimits_{u(M-1)\in U(M-1)} \left[\bar{g}(x(M-1), u(M-1), \omega(M-1))\right], \end{aligned} \tag{5}$$

Step k, for $M-1 > k \geq 0$:

$$J_k(x(k)) := \min_{u(k)\in U(k)} \left[\bar{g}(x(k), u(k), \omega(k)) + J_{k+1}(f(x(k), u(k), \omega(k)))\right] \tag{6}$$

End.

The optimal control policy is then any minimizer of (5) and (6).

4 Numerical Dynamic Programming

4.1 Standard State Space Discretization

The standard method to convert a Dynamic Program into a finite computation problem is to use state space quantization and function interpolation [8, 9]. The state space is quantized into a finite grid

$$x \in \{\eta_1, \eta_2, \ldots, \eta_L\}\,, \tag{7}$$

and at each step of the Dynamic Programming algorithm, the function $J_k(x(k))$ is determined at a finite number of points, $\{\eta_1, \ldots, \eta_L\}$. The function $J_k(x(k))$ at an arbitrary point is then approximated by linear interpolation. In general, a successful approximation of this type of discretization depends upon 'consistency'. This means that a solution closer to a continuous optimal solution can be achieved as the discretization becomes finer [8], which in turn imposes increased computational burden.

Spatial discretization yields the following general step of the Dynamic Programming algorithm:

Step k, for $M-1 > k \geq 0$, and for $1 \leq i \leq L$:

$$J_k(\eta_i) := \min_{u(k)\in\mathcal{U}(k)} \left[\bar{g}(\eta_i, u(k), \omega(k)) + \hat{J}_{k+1}(f(\eta_i, u(k), \omega(k)))\right], \tag{8}$$

where $\hat{J}_k$ is defined by interpolating $\{J_k(\eta_1), \ldots, J_k(\eta_L)\}$.

To check the computational complexity, the above program was setup in MATLAB, with a static TWC model (static emissions conversion efficiency as a function of feedgas air-fuel ratio) and a one state (NO_x storage level) LNT model as the exhaust aftertreatment system. The state was discretized as $0.15 \times \{0, 0.1, \ldots, 0.4, 0.5, 0.7, 0.9, 1\}$ (the maximum trap capacity of the LNT was set to be 0.15 g), and the European Drive Cycle (Euro-cycle), shown in Fig. 2, was used as a drive test cycle. The cycle was sampled at the rate of one second, and the engine speed and load required to follow the cycle at each time step were computed from the model using a gear shift strategy mandated by the Euro-cycle. The minimization in (8) was performed with the MATLAB Optimization Toolbox, using 'constr.m', for $\mu \in \{0, 5, 10, 20, 40, 80\}$.

The total computation time on a Pentium II, 200 MHz PC was roughly 60 hours. This is unacceptable because the engineer needs to be able to evaluate many different parameter values for the LNT model, for example, and in addition, it was deemed important to include the TWC oxygen storage dynamics. Including a second state would result in approximately a half month of computation time. Hence, to reduce the computation time, a new approximation is introduced.

4.2 Approximation via Local Engine Calibrations

The biggest time sink in the optimization process is the minimization operation performed by 'constr.m'. The DISI engine model is nonlinear, and results in many local minima. The idea of the following approximation is to replace the DISI engine model with a finite set of model behaviors, called *calibrations*, parameterized by engine speed and load. More precisely, at each engine speed and load point, the engine model is replaced by a finite set of possible feedgas characteristics, chosen in a way that they are likely to be useful in finding an approximate optimal policy. For the use of calibrations to develop "fixed structure" policies for complex DISI and hybrid diesel powertrains, see [10].

- Quantize the engine speed and brake torque values by a finite grid:

$$N \in \{\psi_1, \psi_2, \ldots, \psi_r\} \text{ (RPM)} \tag{9}$$

$$T_b \in \{\varphi_1, \varphi_2, \ldots, \varphi_l\} \text{ (Nm).} \tag{10}$$

- For each of the points (ψ_i, φ_j), a *normal calibration* is generated by minimizing the cost that represents the weighted sum of fuel consumption and NO_x emissions into the LNT

$$J = \text{fuel} + \nu \cdot NO_{x,lnt}, \tag{11}$$

for $\nu \in \{0, 2, 5, 10, 30, 60, 80, 150\}$, over the engine input parameters throttle position, fuel flow, EGR percent and spark. The $NO_{x,lnt}$ is computed by assuming that the TWC is in steady state, that is, by multiplying the feedgas NO_x emissions by the static NO_x conversion efficiency of TWC. The EGR percent is constrained to be between 0 and 30 for stratified, and 0 to 10 for homogeneous mode, and spark between 5 and 45 degrees before top dead center. The stratified and homogeneous regimes are treated separately during the optimization. Additional constraints are imposed that limit the intake manifold pressure between 5 and 100 KPa, brake torque equal to be φ_j, and engine speed equal to ψ_i, where $\varphi_j \in \{0, 6.25, 15, 25, 35, 45, 55, 65, 75, 85, 95, 105\}$, and $\psi_i \in \{600, 1250, 1750, 2250, 2750, 3250\}$.

For rich operation, the DISI engine model is used to generate a *purge calibration*. This is obtained by maximizing CO emissions into the LNT. It is also assumed that TWC is in steady state, and the mass of CO into the LNT is computed by multiplying the feedgas CO emissions by the static CO conversion efficiency of TWC. Since purge can only take place under rich conditions, the air-fuel ratio is constrained to be less than stoichiometry, and the combustion regime to be homogeneous.

Over the drive cycle, engine output parameters are generated by interpolating calibrations of grided operating points (10) around the true operating point. Figure 3 compares the results of performing the Dynamic Programming with the engine calibrations versus the full optimization over the engine input parameters. This figure plots the tailpipe NO_x emissions in g/km versus fuel economy in miles per gallon, over the Euro-cycle. It is seen that the results are very close. The time taken for generating the set of calibrations was roughly 4 hours (Pentium II, 200 MHz PC). However, once the calibration is done, the Dynamic Programming with different system parameters of the LNT model can be easily and quickly performed because a calibration can be repeatedly used due to its independence of the LNT.

4.3 Vectorization for Multi-State Models

The next step in developing Dynamic Programming as a realistic tool for tradeoff analysis was to consider models with more than one state. This would allow the consideration of important physical phenomena such as oxygen storage in the TWC and the temperature evolution of the aftertreatment elements, which in turn, exponentially increases computations. Using the method based on calibrations, and considering a one state model consisting of static TWC and the dynamic LNT NO_x level studied in Section 4.2, the discretized Dynamic Programming algorithm resulted in a computation time of 5 hours. It was determined that the major computation bottle neck during Dynamic Programming was the interpolation operation (recall (8)). However, this can be remedied by interpolating on a vector scale. The basic idea is to build look-up tables for the dynamic update of the state x, and instantaneous cost $\bar{g}$, as a function of the quantized state η_k, input parameters u, weight μ, engine speed and load. Once these tables are loaded, they are 'vectorized' and used to update (8) on a vector scale. The time spent, based on calibrations generated in Section 4.2,

Table 1: Time consumption on Dynamic Programming based on calibration. The Pentium II, 200 MHz PC was used for computation. To obtain total time consumption, time taken for calibration (4 hours) should be added.

aftertreatment system model	time taken (pointwise)	time taken (vectorized)
one state - static TWC - dynamic LNT	5 hours	20 minutes
two state - dynamic TWC - dynamic LNT	60 hours	40 minutes

is summarized in the Table 1. It is seen that the 'vectorized' Dynamic Programming significantly reduces the time consumption, which enhances the feasibility of optimization for the multi-state models.

5 Case Studies

This section considers several practical case studies that illustrate how design decisions can be made on the basis of optimization. The optimization is based on a static DISI engine model, and a two state, dynamic model of the aftertreatment system. The dynamics of the TWC was limited to the oxygen storage phenomenon since this is crucial for purging. The LNT model is represented by the NO_x storage level. The state space is discretized as x_Θ (fraction of oxygen sites occupied in TWC) $\times$ x_ρ (LNT NO_x storage level in grams):

$$x_\Theta = \{0, 0.25, 0.5, 0.75, 1\} \tag{12}$$

$$x_\rho = C_{lnt} \times \{0, 0.1, \ldots, 0.5, 0.7, 0.9, 1\}, \tag{13}$$

and the maximum capacities of TWC (C_{twc}) and LNT (C_{lnt}), were set to 0.5 g and 0.15 g, respectively. The optimization was done with the interpolated DISI engine calibrations over a range of μ. As an example, Fig. 4 shows the normalized optimal trajectories of TWC oxygen storage level and LNT NO_x storage level, with NO_x emissions constrained to Stage IV NO_x Emissions Standard (0.08 g/km) of the Euro-cycle. It is seen that the purging process is delayed by a few seconds until the oxygen stored in the TWC is mostly released. This is because CO is oxidized before it reaches the LNT, due to excess oxygen released from the TWC. Thus, in order for CO to reach the LNT to purge the stored NO_x, the oxygen level of the TWC must first be brought down to low levels. The figure also captures the unsolicited NO_x release in the high vehicle speed portion of the Euro-cycle. This release is due to high engine speed and load conditions, resulting in high LNT temperature.

5.1 Case Study 1: TWC and LNT Capacities

The capacity of the LNT used on a vehicle will be determined by a tradeoff between manufacturing price and system performance. To study this tradeoff, optimal solutions are obtained with various maximum trap capacities for the LNT:

$$C_{twc} = 0.5, \quad C_{lnt} \in \{0, 0.15, 0.5, 1, 2\}. \tag{14}$$

The fuel economy in miles per gallon, for the Stage IV NO_x Emission Standard of the Euro-cycle, is shown in Fig. 5 as a function of maximum trap capacity of the LNT. It is seen that fuel economy improvement rapidly rolls off as trap capacity increases, and is mostly improved at low maximum trap capacity. In particular, when C_{lnt} is equal to zero, the LNT is virtually removed, leaving TWC as the unique component in the aftertreatment system. In this case, the DISI engine mostly operates in stoichiometric mode so that the TWC maintains high NO_x conversion efficiency over the cycle, thereby, significantly increasing fuel consumption.

The effect of TWC oxygen storage capacity on fuel economy is also evaluated. The maximum oxygen storage capacity of TWC was varied over

$$C_{twc} \in \{0.25, 0.5, 1.5, 2\}, \quad C_{lnt} = 0.15. \tag{15}$$

Figure 6 shows the fuel economy as a function of maximum capacity. As can be seen, fuel economy decreases as maximum capacity of TWC increases. This is because purging is delayed until reductants, such as CO, HC and H_2, are effectively delivered to the LNT, and the delay is proportional to the emptying time of the oxygen stored in the TWC.

5.2 Case Study 2: Removal of Homogeneous Lean Mode

For the engine under study, the homogeneous lean mode is limited to air-fuel ratios from 15 to 20. The removal of the homogeneous lean mode is considered in order to simplify the engine operation and control strategy. The effect of removal is evaluated by Dynamic Programming, and the fuel economy and tailpipe NO_x emissions over the Euro-cycle are shown in Fig. 7. The figure shows that the loss of fuel economy without the homogeneous lean mode is 0.3 miles per gallon, which corresponds to a 0.78 % loss, with Stage IV NO_x Emission Standard of Euro-cycle. However, for Stage III NO_x Emission Standard (0.15 g/km), the loss of fuel economy is 1.4 miles per gallon. This is a 3.65 % loss, which is not acceptable. It is seen that the fuel economy is largely degraded during the high vehicle speed region of the Euro-cycle without the homogeneous lean mode. This is because without the homogeneous lean mode, the DISI engine has to operate in the stoichiometric mode since the stratified mode is not viable at high ranges of engine speed and load. Thus, allowing the homogeneous lean mode gives more freedom for the DISI engine to achieve better fuel economy in this region. However, in the limit as the NO_x constraint becomes lower and lower, the engine must be operated in the stoichiometric mode. This explains why for Stage IV NO_x regulation, the homogeneous lean mode is not useful.

5.3 Case Study 3: Effect of Temperature Dynamics

The aftertreatment systems' temperature dynamics is important due to significant dependency of LNT NO_x storage capacity on temperature. Thus, the effect of temperature dynamics on DISI engine performance is studied in this section. In general, the time constant of the LNT temperature dynamics depends on many factors such as the thermal effect of chemical reactions, but here, a reasonable value of 30 seconds is assumed. For Dynamic Programming, the state space of LNT temperature is discretized as follows:

$$x_{temp} = \{273, 440, 507, 540, 574, 607, 641, 674, 707, 774, 874, 1600\} \ \ (\mathrm{K}). \tag{16}$$

Figure 8 shows the Dynamic Programming result. It is seen that the three state model results in roughly 38.9 miles per gallon of fuel economy with constrained NO_x emissions to Stage IV NO_x Emission Standard of Euro-cycle, which is a 1.83 % improvement compared with the two state model, thereby, indicating that the temperature dynamics increases the overall NO_x storage capacity of LNT over the Euro-cycle. Figure 9 plots the normalized optimal trajectories of TWC oxygen storage level and LNT NO_x storage level of three state model, with NO_x emissions constrained to Stage IV NO_x Emissions Standard of the Euro-cycle. It shows frequent purging in the high vehicle speed portion of the Euro-cycle, compared with the simulation result of the two state model shown in Fig. 4. This is because slow temperature dynamics keeps the overall NO_x storage capacity much lower than that of two state model at this portion, and NO_x stored in LNT needs to be purged before it exceeds the storage capacity. In addition, the amount of NO_x owing to unsolicited release is very small because the temperature dynamics prevents an abrupt change of NO_x storage capacity. On the other hand, overall higher NO_x storage capacity over the Euro-cycle enables less frequent purging than two state model at low vehicle speed portion of the Euro-cycle, while tailpipe NO_x is constrained to the same level.

5.4 Case Study 4: Optimal Gear Shift

The fuel consumption and feedgas properties of the DISI engine can be significantly varied by selecting different gear positions. In this section, the effect of gear shift strategy on fuel economy and NO_x emissions is studied by seeking the optimal gear trajectory. Over the Euro-cycle, demanded engine speed and load are determined by gear position and the rotational dynamics (Appendix); see Fig. 10. Hence, when freedom of gear selection is given, the rotational dynamics should be reflected in the Dynamic Programming since engine speed and load cannot be instantaneously changed. One possible choice for the additional state of the system is engine speed. However, engine speed is continuous, and requires coarse discretization over the wide range to apply Dynamic Programming. An alternative comes from the gear position. The gear position determines engine speed and load given vehicle speed from the Euro-cycle, and can have only one of six discrete values (neutral, 1st, ..., 5th gear position), which is favorable for performing Dynamic Programming. Thus, the gear position was chosen as a third state, in addition to TWC oxygen storage level and the NO_x storage level in LNT. The optimization result is shown in Fig. 11. It is seen that the optimal gear shift strategy results in roughly 43.3 miles per gallon of fuel economy with NO_x constrained to Stage IV NO_x Emission Standard, which is 13.35 % improvement over the standard gear shift strategy. This shows that gear shift optimization is an important means for fuel economy improvement.

5.5 Case Study 5: Development of Cycle-independent Control Policy

The optimization problem posed in Section 3 is based on a specific driving cycle, that is, the Euro-cycle. Thus, the optimal control policy obtained from the Dynamic Programming is *cycle-*

dependent, which is unacceptable for vehicle deployment. It is desirable to determine if a *cycle-independent* control policy can be found which achieves comparable performance to that of the optimal policy on the Euro-cycle. One way to achieve this is to obtain the optimal control policies at each point of a set of engine speeds and loads, and then implement the steady-state-optimal policy along an arbitrary driving cycle.

At constant speed and load, the model of the combined engine and aftertreatment systems, (1) and (2), becomes time-invariant, and the infinite horizon optimization problem can be well defined. The objective is to find the optimal control input, $u(k)$, as a function of engine speed and load, that minimizes the average cost functional

$$J(x) = \lim_{K\to\infty} \frac{1}{K} \min_{u\in U_l} \sum_{k=0}^{K-1} g(y(k), u(k)) = \lim_{K\to\infty} \frac{1}{K} \min_{u\in U_l} \sum_{k=0}^{K-1} \bar{g}(x(k), u(k)), \tag{17}$$

where U_l represent constraints for u imposed by meeting constant engine speed and load. The functions $g(y(k), u(k))$ and $\bar{g}(x(k), u(k))$ are instantaneous costs, which are weighted sums of fuel consumption and tailpipe NO_x emissions out of the LNT, as defined in (3). Over a range of weight μ, a solution can be obtained via Dynamic Programming [8]:

Step k, for $k > 0$:

$$J_k(x(k)) := \min_{u(k)\in U_l} [\bar{g}(x(k), u(k)) + J_{k-1}(f(x(k), u(k)))] \tag{18}$$

End if $J_k(x(k))/k$ has converged.

Thus, if the average cost (17) converges, the (stationary) optimal solution can be obtained at the end of iterations [8]. For numerical Dynamic Programming, the methods discussed in Section 4 can be employed to speed up the computation.

The solutions obtained from the infinite horizon optimization at a set of constant engine speed and load were scheduled along the Euro-cycle, and the fuel consumption and tailpipe NO_x emissions were computed via simulations. The performance curves for a range of μ is shown in Fig. 12 with that of the finite horizon optimal policy as a target.

It is seen that the fuel economy of the scheduled infinite horizon solution results in 37.2 miles per gallon of fuel economy, with NO_x constrained to Stage IV NO_x Emission Standard. This is a 2.6 % loss when compared to the finite horizon optimal solution, which has 38.2 miles per gallon of fuel economy. As an example, Fig. 13 plots the normalized trajectories of TWC oxygen storage level and LNT NO_x storage level of scheduled infinite horizon solution, with NO_x emissions constrained to Stage IV NO_x Emissions Standard of the Euro-cycle. It shows that purging patterns at low vehicle speed portion of the cycle resemble those of the finite horizon optimal solution in Fig. 4. However, the overall NO_x storage level is kept lower than that of finite horizon optimal solution at high vehicle speed portion, thereby, consuming more fuel.

6 Conclusions

In this paper, a problem of predicting the best emission constrained fuel economy of a direct injection spark ignition powertrain over a drive cycle was investigated. This problem is difficult because the search for the optimal trajectory has to be done over all possible trajectories of the engine and the aftertreatment on a drive cycle. The search procedure is based on the Dynamic Programming algorithm. The procedure is made computationally tractable by combining several ideas that involve (i) model simplification; (ii) state and control discretization; (iii) restricting the search to a smaller set of trajectories that, based on engineering judgment, are deemed likely to contain the optimal policy, and (iv) careful treatment of computer implementation details. Numerical results have demonstrated significant reduction in the computation time, while near optimal solutions are generated.

The procedure has been used in several case studies where the effect of adjusting hardware parameters or control strategy on the fuel economy was evaluated. The ability to conduct this kind of assessments is very important early in the development cycle of an automotive system and its control strategy.

This study resulted from a cooperative research project between researchers from Ford Research Laboratory and researchers from the University of Michigan. It demonstrates how advanced optimization techniques can be adapted to a realistic industrial problem.

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Appendix

Demanded Brake Torque

The engine speed is determined by the vehicle speed, S_v (km/h), and the overall drive ratio (R), which is a function of drive ratio (g_r) from the crank shaft to the wheel, and the radius (m) of the tire (r_t):

$$N = \frac{60 S_v}{3.6 \times 2\pi R} \quad \text{(A-1)}$$

$$R = \frac{r_t}{g_r}. \quad \text{(A-2)}$$

In this study, the radius of the tire is set to 0.31 m. The drive ratio, g_r, is given as shown in Table A-1, as a function of gear position. When vehicle speed is zero, the gear is in neutral position and the engine speed is set to idle speed, which is, 625 RPM.

Table A-1: Drive ratio from crank shaft to wheel.

Gear position	1st	2nd	3rd	4th	5th
Drive ratio	14.48	7.87	5.57	4.16	3.10

The load torque, T_l (Nm), can be obtained from the engine speed and the road-load power, P_r

(Watt) [1]:

$$P_r = (2.73\, C_R M_v + 0.0126\, C_D A_v S_v^2) S_v \tag{A-3}$$

$$T_l = \frac{60 P_r}{2\pi N} + T_{aux}, \tag{A-4}$$

where C_R, M_v, C_D, and A_v are coefficients of rolling resistance (=0.0095), vehicle mass (=1313 kg), drag coefficient (=0.33), and frontal area of vehicle (=2.05 m^2), respectively. The auxiliary torque, T_{aux}, represents the additional torque required to drive the engine accessories (air conditioner, generator, various pumps, etc.), and are time-varying throughout a driving cycle. For example, the averaged auxiliary torque over the European Drive Cycle is roughly 4.5 Nm. Then, the demanded brake torque to maintain vehicle speed S_v can be determined from the rotational dynamics as:

$$J \frac{2\pi}{60} \dot{N} = T_b - T_l, \tag{A-5}$$

where J is the sum of the engine inertia (=0.2 kg·m^2) and the vehicle inertia (=$M_v \times R^2$ kg·m^2).

Figure 1: Complete model for emission system.

Figure 2: European Drive Cycle for emissions evaluation.

Figure 3: Fuel economy versus NO_x emissions of optimal policy with calibrations and from full optimization, over the Euro-cycle. The DISI engine, TWC models are quasi-static. The LNT NO_x filling and emptying is dynamically updated.

Figure 4: Normalized optimal trajectories of TWC oxygen storage level and LNT NO_x storage level, with NO_x emissions constrained to Stage IV NO_x Emissions Standard. The maximum capacities of TWC and LNT, C_{twc} and C_{lnt}, were set to 0.5 g and 0.15 g, respectively.

Figure 5: Fuel economy satisfying Stage IV NO_x Emission Standard of Euro-cycle with various maximum trap capacity of LNT.

Figure 6: Fuel economy satisfying Stage IV NO_x Emission Standard of Euro-cycle with various maximum oxygen storage capacity of TWC.

Figure 7: Fuel economy and NO_x emissions over Euro-cycle with, and without homogeneous lean mode.

Figure 8: Fuel economy versus NO_x emissions of optimal policy with two state (dynamic TWC oxygen storage, dynamic LNT NO_x storage, static LNT temperature) and three state (dynamic TWC oxygen storage, dynamic LNT NO_x storage, dynamic LNT temperature) model.

Figure 9: Normalized optimal trajectories of TWC oxygen storage level and LNT NO_x storage level of three state model, with NO_x emissions constrained to Stage IV NO_x Emissions Standard.

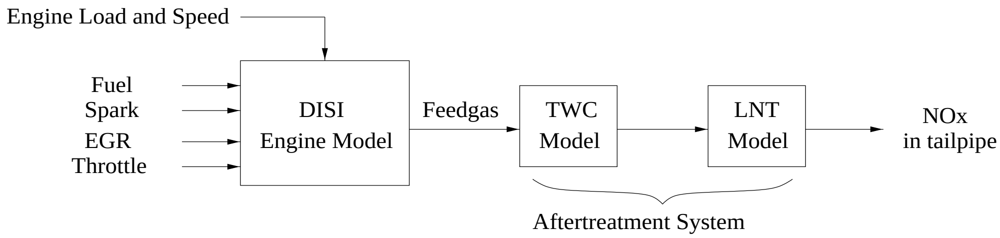
Figure 10: Emission system with free gear ratio.

Figure 11: Performance comparison of standard gear shift and optimally scheduled gear shift.

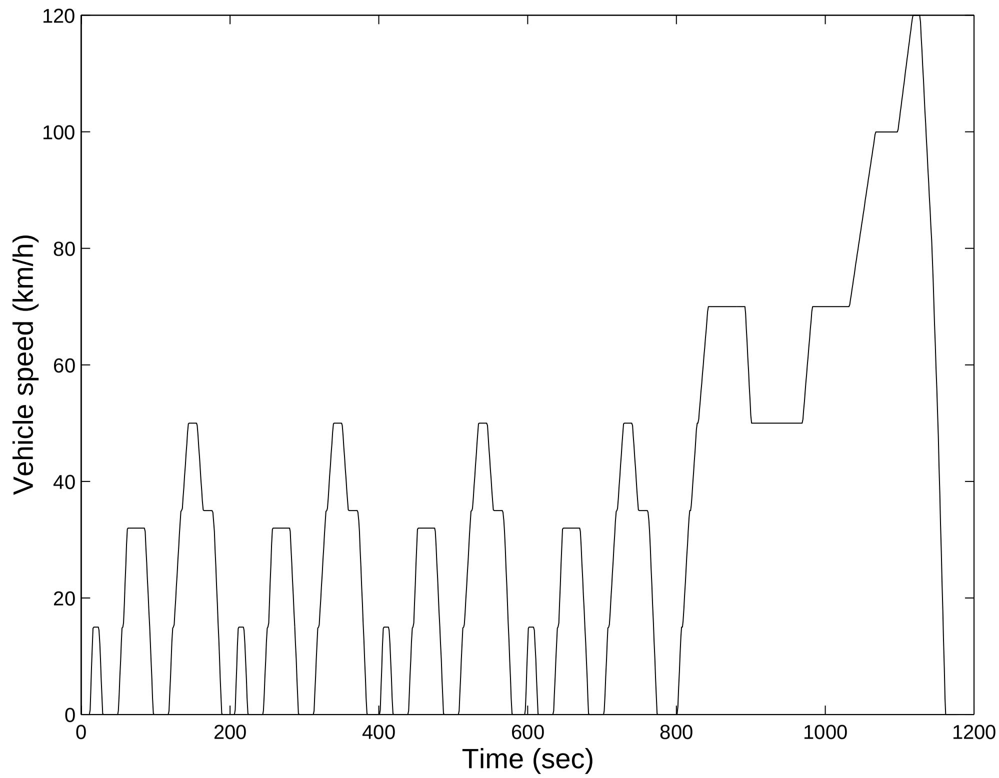
Figure 12: Fuel economy and NO_x emissions over Euro-cycle with scheduled control policy of infinite horizon optimization and optimal policy of finite horizon optimization.

Figure 13: Normalized trajectories of TWC oxygen storage level and LNT NO_x storage level of scheduled infinite horizon solution, with NO_x emissions constrained to Stage IV NO_x Emissions Standard.

Jun-Mo Kang, Ilya Kolmanovsky and J. W. Grizzle: Figure 1



Jun-Mo Kang, Ilya Kolmanovsky and J. W. Grizzle: Figure 2



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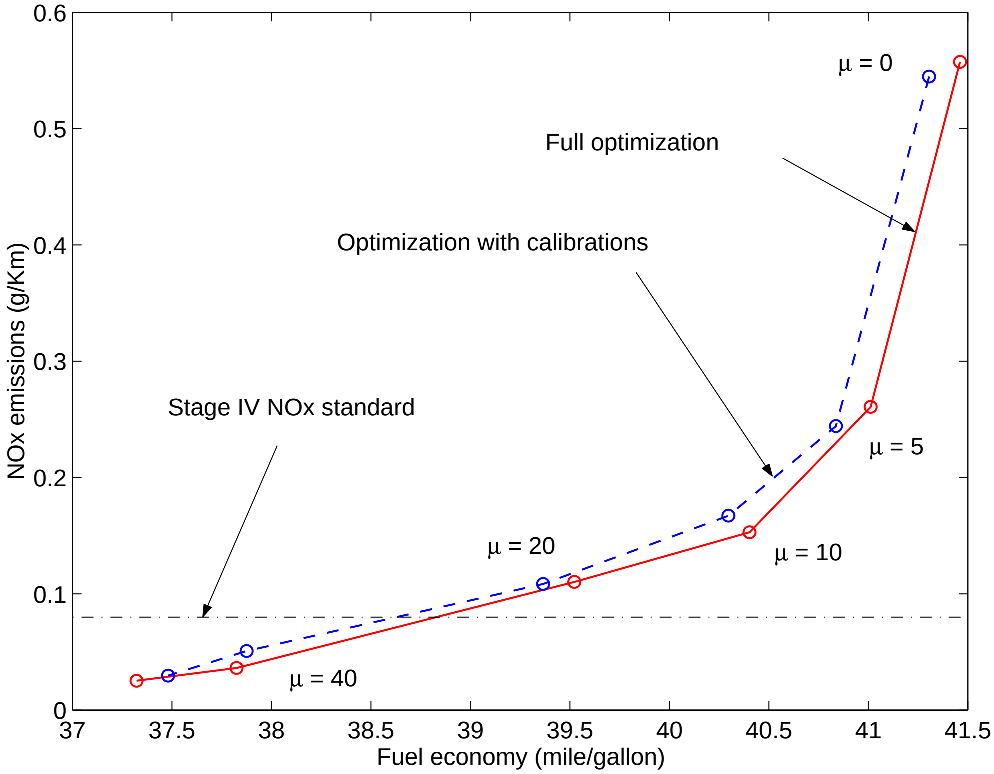

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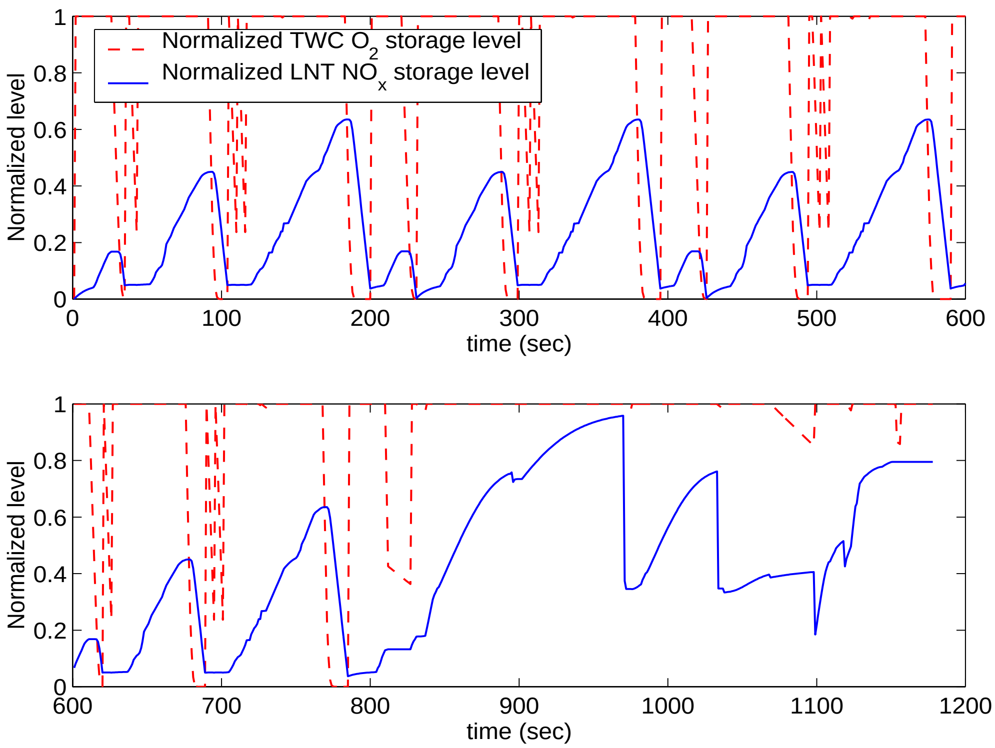

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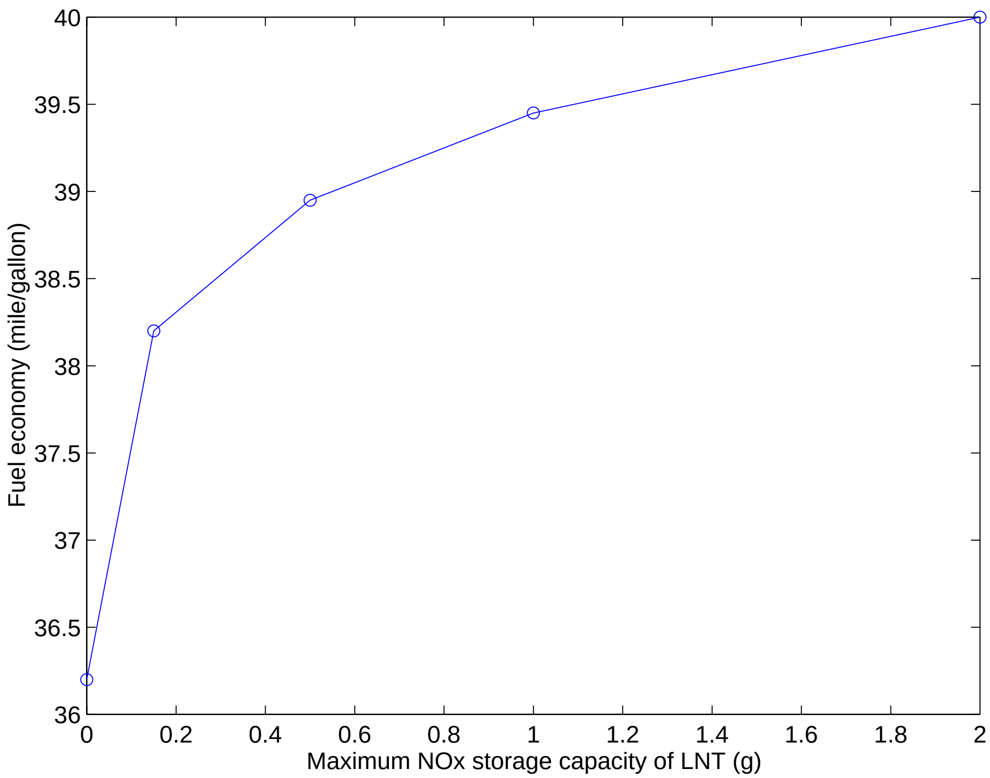

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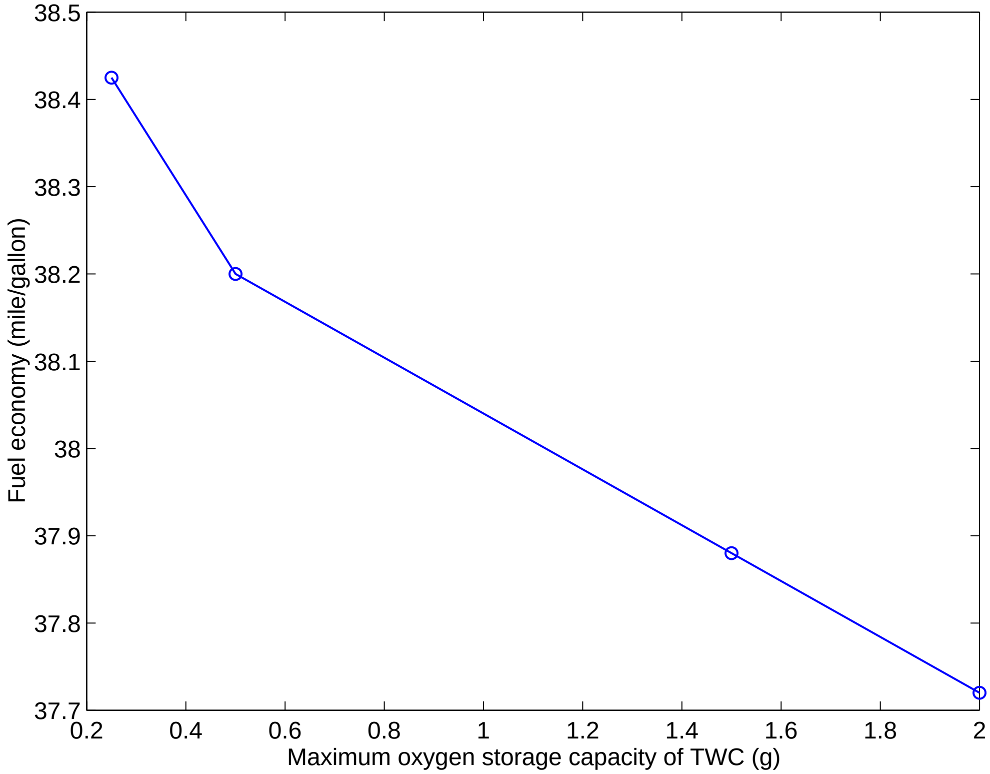

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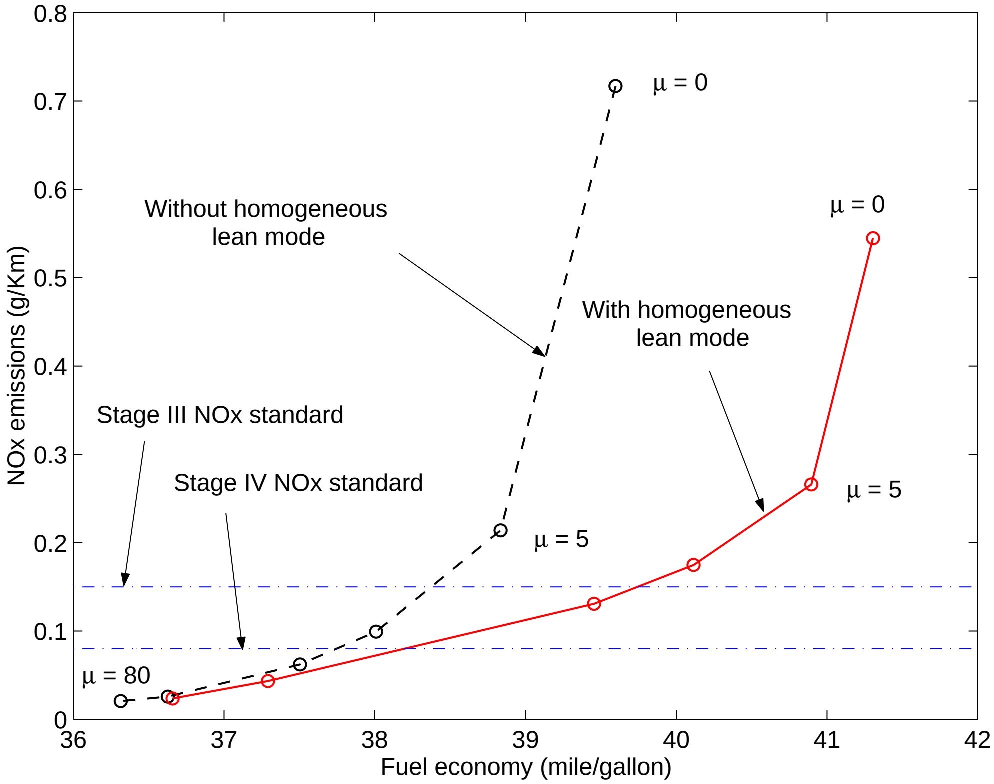

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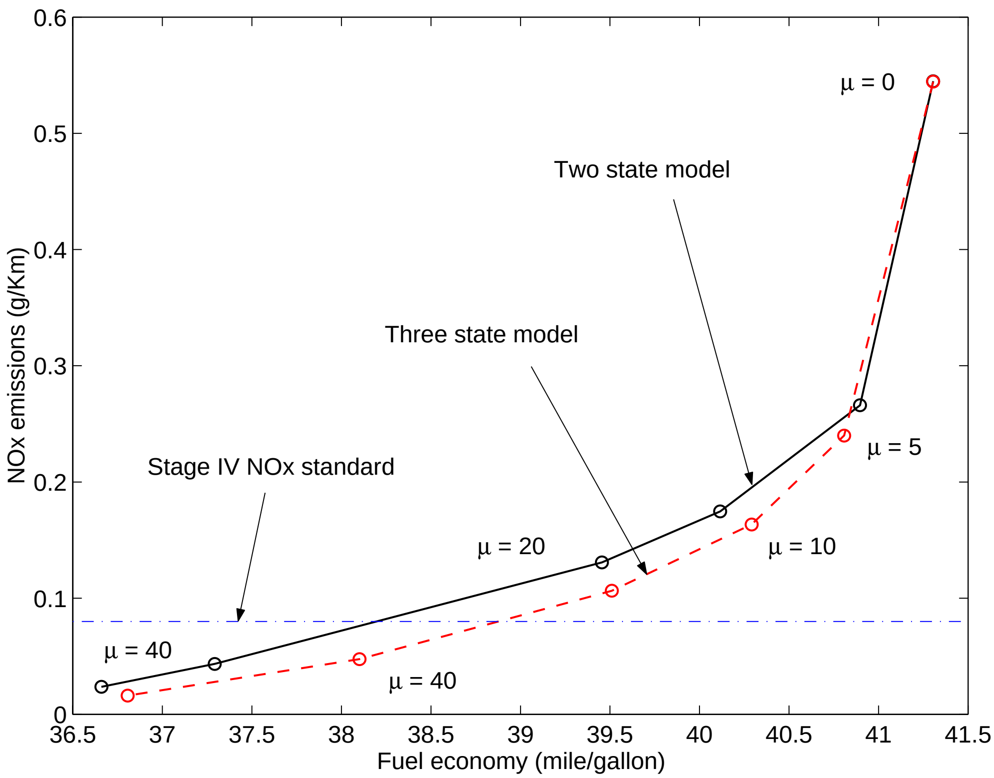

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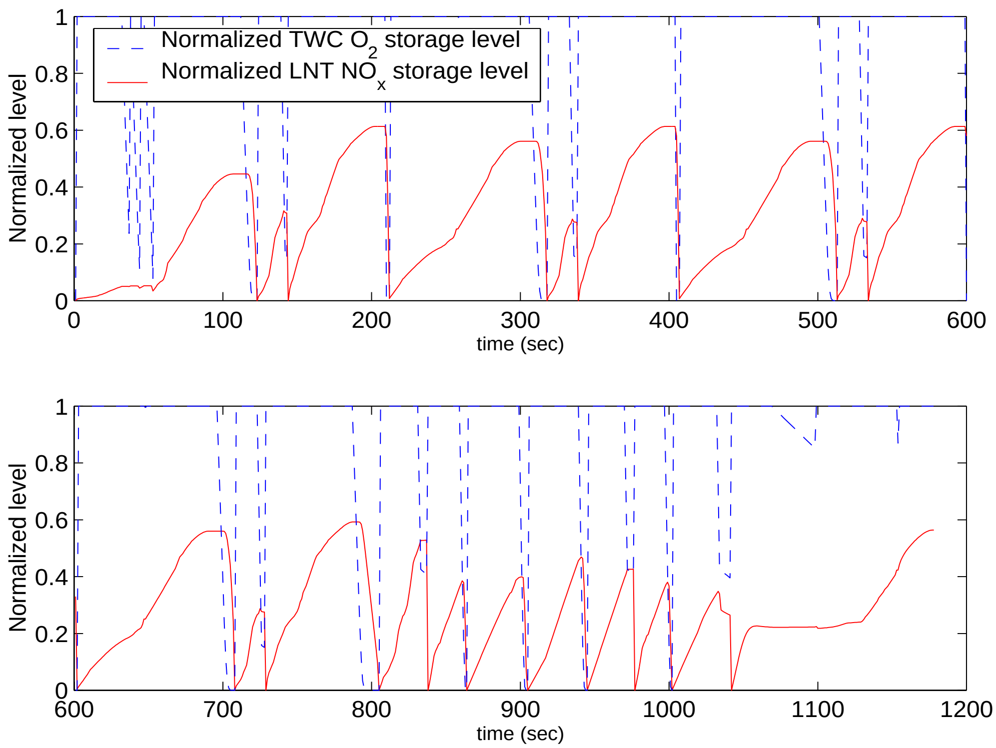

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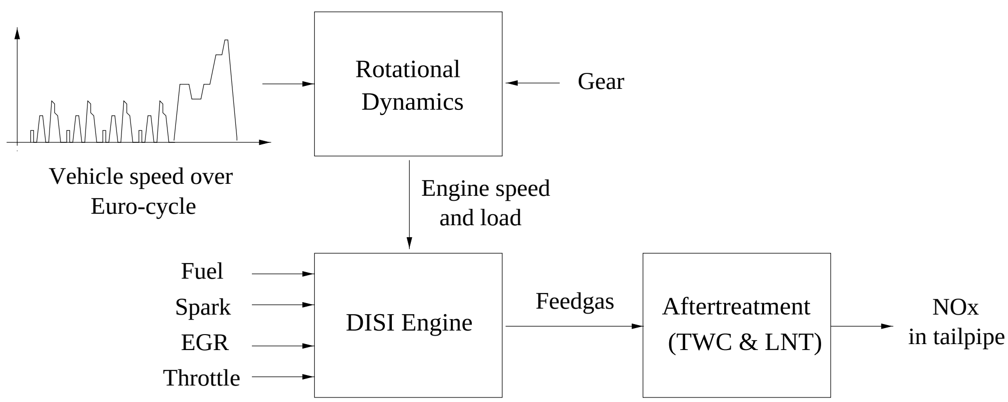

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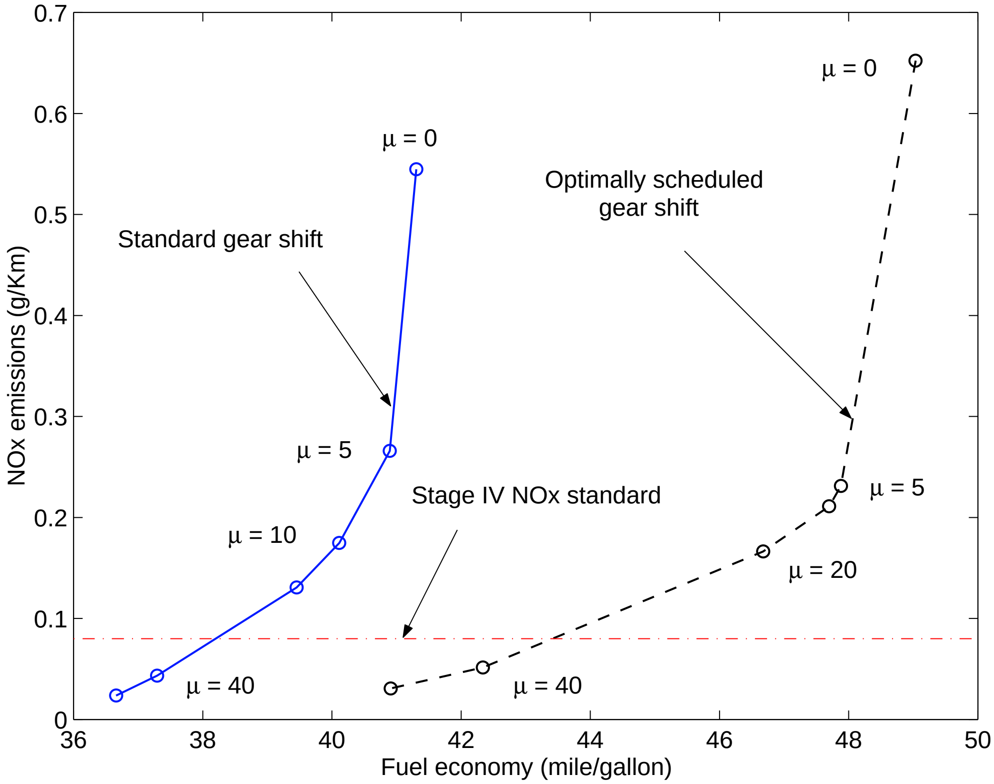

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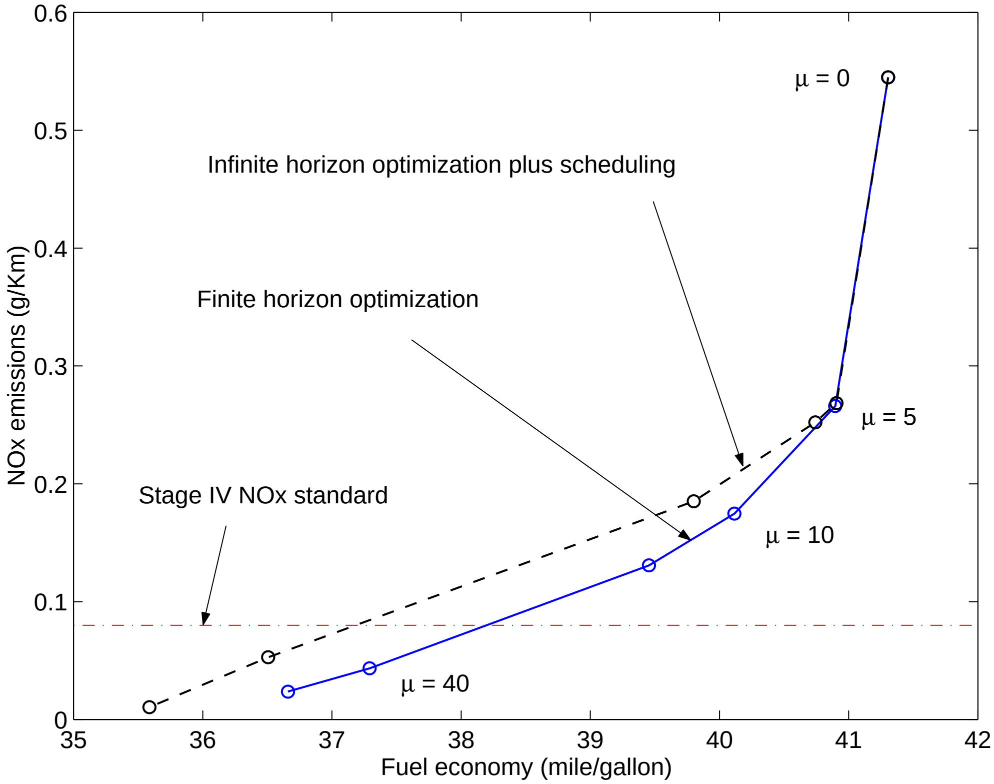

Jun-Mo Kang, Ilya Kolmanovsky and J. W. Grizzle: Figure 13