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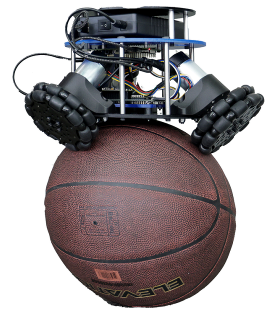
Summary of our approach with ROB 201

I condensed Calculus I, Calculus II, and Ordinary Differential Equations into a single 4-credit semester course by building it around three major projects: numerical integration, constrained optimization, and feedback control of the ROB 311 BallBot (pictured). These projects provided a natural structure—giving the course a clear beginning, middle, and end—while also ensuring that students engaged deeply with fundamental mathematical concepts in a practical, application-driven way.



Rather than following the traditional calculus sequence, I deconstructed the subject and rebuilt it from the ground up, rearranging topics to improve conceptual flow. Instead of starting with derivatives, I introduced students to numerical integration first, leveraging their intuitive understanding of summation (I thank Prof. Chad Jenkins for this idea). Antiderivative rules were separated into a distinct chapter, reducing the frustration of manual symbolic integration. This restructuring allowed calculus to unfold in a way that aligned with how students naturally think about and use mathematics in engineering applications.

To make the subject more engaging and relevant, I shifted the emphasis from manual computation to computational problem-solving. By integrating software tools, students could visualize mathematical concepts and experiment with them interactively. Jupyter notebooks played a crucial role in this transformation, allowing homework assignments to combine explanatory text with executable code. This approach harnessed students' intrinsic motivation—when their code produces meaningful results, they are more willing to engage with the underlying mathematical theory.

I also identified natural synergies across topics. For example, numerically evaluating an integral and solving an ODE share the same computational framework. By introducing numerical integration early, I unknowingly prepared students for ODEs without them realizing it. Similarly, my second project, which used constrained gradient descent to model a 3-meter platform diver's motion, was actually an ODE problem in disguise. Students were solving for trajectories without explicitly recognizing the underlying differential equations, making the eventual transition to ODEs much smoother. This deliberate layering of concepts ensured that students were ready for ODEs by the time they formally encountered them.

By prioritizing real-world applications, computational exploration, and a logical restructuring of topics, I created a course that not only condenses three traditional courses into one but also makes calculus more intuitive, engaging, and effective for engineering students.

Thank you for your time,

Jessy Grizzle

Jessy Grizzle
Jerry and Carol Levin Prof. of Engineering, Elemer Gilbert Distinguished University Professor,
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Topic Coverage by the Traditional Paradigm

In the following, we re-organize the Table of Contents of Calculus for the Modern Engineer into the more traditional Calculus I through IV groupings[3]. The textbook does not cover infinite series and their associated convergence tests, multivariable integration and its associated topics of Green's functions, divergence, and curl, nor linear algebra, the topic of ROB 101 *Computational Linear Algebra*. It is hoped that this layout will allay fears that whole swaths of Calculus have been gutted (other than the aforementioned topics). We encourage students to take a traditional Multivariable Calculus course for the missing material.

About the Course

1. Philosophy of the Course
2. Introduction
3. Julia and LLM/GenAI Resources
4. Got Calculus Dread?
5. Resources for a Traditional Approach to Learning Calculus
6. Julia-based Sources Recognizing the Important Role of Programming in Bringing Math to Life

Pre-calculus (Mostly for Student Review, Treated in HW 1)

1. Notation or the Language of Mathematics
2. The Approximation Principle: The Essence of Calculus through the Lens of Irrational Numbers
3. Algebraic Manipulation and Inequalities
4. Functions, Domains, Ranges, Inverses, and Compositions
5. Strictly Monotonic Functions and Relation to Existence of Inverse Functions
6. Trigonometric and Inverse Trigonometric Functions
7. 3-Link Manipulator
8. Powers and Roots for Integer and Rational Exponents
9. Real Exponents
10. Exponentials and Logarithms
11. Euler's Formula
12. Hyperbolic Trig Functions and Relation to Euler's Formula
13. Binomial Theorem
14. (Optional Read:) Proofs Associated with the Chapter

[3]Calculus III, Multivariable Calculus, is only represented through partial derivatives and Jacobians.

Calculus I: Differentiation

1. Theoretical Notions
    (a) Vocabulary and Helpful Notation
    (b) What is a Mathematical Proof? And Why are Mathematical Proofs Important?
    (c) Countable Sets
    (d) Proofs by Induction
    (e) Maximum and Minimum Values of Sets and Their Generalizations
    (f) Maximum and Minimum Values of Functions and Their Generalizations
2. Specific Finite Sums, Geometric Sums, and Their Applications
3. Limits at Infinity
    (a) Rational Functions
    (b) Products and Ratios of Exponential and Monomial Terms
4. Finite Limits
    (a) Intuition for Limits from the Left and the Right
    (b) Formal Definition of Limits from the Left and the Right
    (c) Two-sided Limits and Continuous Functions
    (d) The Power of Continuity Done Right: Key Properties of Continuous Functions or Why We Care about Continuity
    (e) Limit Algebra
    (f) The Squeeze Theorem
    (g) L'Hôpital's Rule for Evaluating Limits (taught after differentiation, of course)
5. Differentiation
    (a) The Derivative as the Local Slope of a Function
    (b) The Derivative as a Local Linear Approximation of a Function
    (c) Software Tools
        i. Symbolic Differentiation
        ii. Numerical Differentiation
        iii. Automatic Differentiation
        iv. Guidelines on Choosing Differentiation Methods
    (d) Differentiation Rules that all Engineers are Expected to Understand
        i. Derivative of a sum
        ii. Product Rule
        iii. Ratio Rule
        iv. Chain Rule
    (e) Use Cases of the Single-variable Derivative
        i. From Position to Velocity
        ii. If the Derivative does not Change Sign, the Function is Monotonic
        iii. Extreme values of functions
        iv. Taylor's and Maclaurin's Polynomials and Series
    (f) Typically covered in Calc III, but part of Differentiation due to ROB 101 being a Pre-requisite
        i. Partial Derivatives
        ii. Packaging Partial Derivatives to form Jacobians, Gradients, and Hessians
        iii. The Total Derivative or the Chain Rule on Steroids

(g) Engineering Applications of the Derivative
    i. Root Finding (Vector-valued functions)
    ii. Minimization without Constraints (multivariable, scalar-valued)
        A. Gradient Descent Algorithm
        B. Second Derivative Tests for Local Min and Max
    iii. Lagrange Multipliers and Constrained Optimization
        A. Motivating Problems and Vocabulary of Constrained Optimization
        B. Lagrange Multiplier for a Problem with a Single Equality Constraint
        C. (Optional Read:) Lagrange Multipliers for a Problem with a Vector of Equality Constraints
        D. (Optional Read:) Proof behind Lagrange Multipliers for Equality Constraints
        E. 3-Link Manipulator Meets Inequality Constraints
    iv. Dynamics à la Lagrange
        A. Kinetic and Potential Energy
        B. Symbolic Computational Tools
        C. Lagrange's Equations

6. (Optional Read:) Proofs Associated with the Chapter

Calculus II: Integration

1. The Riemann Integral (aka, Riemann-Darboux Integral)
    (a) Simple Version of Riemann Lower and Upper Sums
    (b) Riemann Integral of a Continuous Function over a Closed Bounded Interval
    (c) Illustration of the Riemann Indefinite Integral of a Monomial
    (d) Are all Functions Riemann Integrable?
2. Properties of the Riemann Integral
    (a) A Basic Additivity Property of Area Under a Curve
    (b) Integrating Linear Combinations
    (c) Making Sense of an Integral when its Lower Limit is Greater than its Upper Limit
    (d) Generalized First Additivity Property of Integrals
    (e) Dummy Variables of Integration
    (f) Shifting and Integration
    (g) Scaling and Integration
3. Numerical Methods for Approximating Riemann Integrals
    (a) Trapezoidal Rule
    (b) Simpson's Rule
    (c) Julia Packages
    (d) (Optional Read:) Even and Odd Functions as Great Test Cases
4. Applications of the Definite Integral
    (a) From Speed to Change in Position
    (b) Ballistic Motion
    (c) Area between Two Functions
    (d) Modeling the Links in a Robot
    (e) Solids of Revolution or the Surprising Power of the Rectangle

(f) Path Length or Arc Length
(g) (Optional Read:) Derivation of the Center of Mass Equations for a Continuous Object

5. Fundamental Theorems
    (a) Uniting Integration and Differentiation through the Fundamental Theorems of Calculus
    (b) Using the Second Fundamental Theorem of Calculus for Definite Integration
        i. The Art of the Antiderivative: Inverting Differentiation Rules to Find Antiderivatives
        ii. The Fundamental Rule: Integrating the Differential
        iii. Inverting the Chain Rule: Integration by Substitution, aka u-Substitution
        iv. Inverting the Product Rule: Integration by Parts
        v. Trigonometric Substitutions for Radicals
        vi. Antiderivatives of Rational Functions by Partial Fraction Expansion (PFE)
6. Why Conflating Integration and Antiderivatives is a Pedagogical Pitfall in Calculus
7. Unbounded Limits of Integration or Unbounded Functions
    (a) Type-I Improper Integrals: Unbounded Limits of Integration
    (b) Comparision Test
    (c) Absolute Integrability
    (d) Type-II Improper Integrals: Vertical Asymptotes
8. (Optional Read:) Proofs Associated with the Chapter

Calculus III: Multivariable Calculus

1. Jacobians, gradients, Hessians
2. Nothing more. This is a deliberate choice.

Calculus IV: ODEs

1. Introduction
    (a) Let's Start Simple: One equation, One Unknown, One Derivative
        i. Analytical Solutions via Antiderivatives (Separation of Variables and Integrating Factors)
        ii. Numerical Solutions
        iii. Finite Escape Time
        iv. One-Dimensional ODE with Multiple Solutions
    (b) (Optional Read:) Can an ODE Have No Solution?
    (c) (Optional Read:) The Independent Variable in an ODE can be any Strictly Increasing Quantity
2. More Complex ODEs
    (a) Higher-order ODEs and a Direct Current (DC) Motor Model
    (b) Vector ODEs: Multidimensional Dynamics, Single Independent Variable
    (c) The Return of the Robot Equations
3. What is a Solution to a First-order Vector ODE ($\dot{x} = f(x)$)
4. Existence and Uniqueness of Solutions
5. ODE Examples from a Plethora of Engineering Domains

6. Solving ODEs via Software
7. Linear Systems of ODEs
   (a) Examples from Circuits and Mechanics
   (b) Higher-order Linear ODEs
   (c) Linearization of Nonlinear ODE Models
   (d) The Matrix Exponential
   (e) Software Tools for Linear ODEs
   (f) Properties of Solutions to Linear ODEs
   (g) Eigenvalues and Eigenvectors to the Rescue
   (h) Exponential Stability of Linear Systems of ODEs with Implications for Nonlinear ODEs
8. Euler's Method
9. Resonance in ODEs
10. (Optional Read:) Proofs Associated with the Chapter

Calculus IV: Laplace Transforms

1. Setting the Stage for Developing Laplace Transforms in the Context of Feedback Control
   (a) Single-Input Single-Output (SISO) Linear Systems
   (b) Input-Output and State-Variable Models
   (c) Input-Output Models from Circuits
   (d) A State-Variable Model of a (Planar) Segway Transporter
2. The Laplace Transform and an Algebraic Approach to Linear ODEs
   (a) Definition and a Key Property
   (b) Common Laplace Transform Pairs
   (c) (Optional Read:) How to Compute Laplace Transform Pairs by Hand
   (d) Software Tools
   (e) Transfer Functions
   (f) The Segway Transporter à la Laplace
3. Poles, Zeros, and BIBO Stability
4. Unity Feedback Systems
   (a) Closed-loop Transfer Functions
   (b) Two Common Compensators: Proportional (P) and Proportional-Derivative (PD)
5. Performance Specifications
   (a) Steady-State Error
   (b) Transient Response of First- and Second-order Systems
6. Relating Transient Response to Poles and Zeros
   (a) First-order System without a Zero
   (b) Second-order System without a Zero
   (c) Effects of Zeros

7. Design of Cascade Compensators and Pre-compensators
   (a) First-order Systems
   (b) Second-order Systems
   (c) Remarks on Dealing with More Complicated Systems
8. Feedback Design for a Linearized Model of a Planar Segway Transporter
   (a) Controlling Body Lean Angle
   (b) Controlling Speed and Lean Angle
   (c) The Real Deal: Implementing the Controller on the Nonlinear Model
   (d) (Optional Read:) Robot Equations for a Planar Segway