Walking & Running in Bipedal Robots: Control Theory and Experiments

EECS Department
University of Michigan

Jessy W. Grizzle

This work is supported by NSF grant ECS-0322395

Acknowledgements

Gabriel Abba
(Metz, France)

ROBEA

(French National Project)

Franck Plestan
(Nantes, France)

Christine Chevallereau
(Nantes, France)

Carlos Canudas-de-Wit
(Grenoble, France)

Gabriel Buche
(Grenoble, France)

Yannick Aoustin
(Nantes, France)

Acknowledgements

Carlos Canudas-de-Wit
(Grenoble, France)

ROBEA

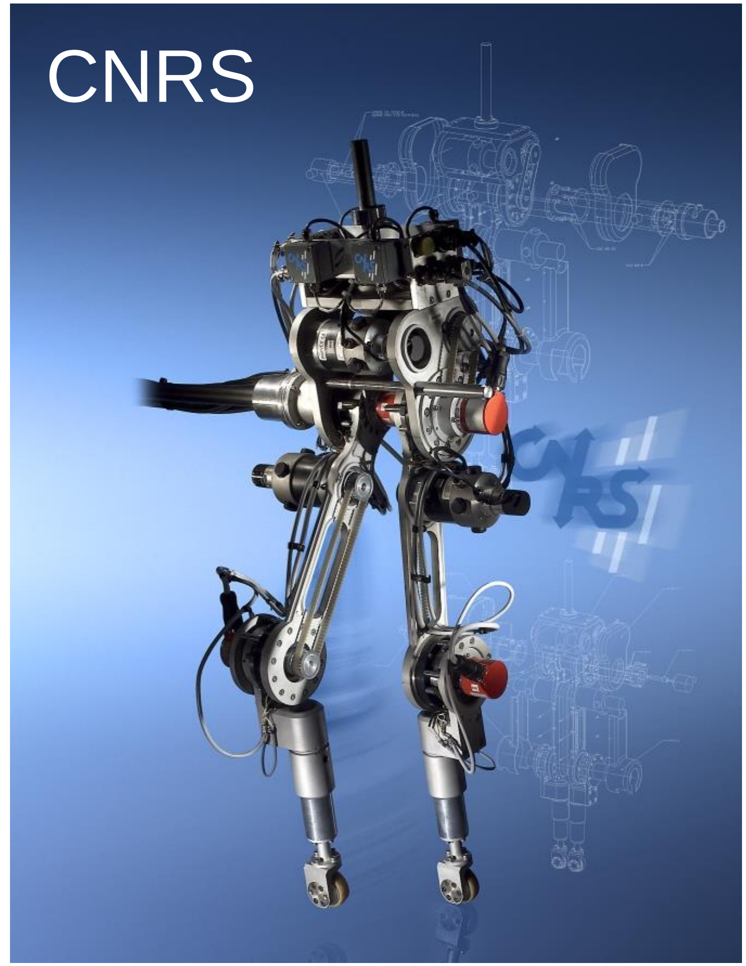
(A French National Project)

Christine Chevallereau
(Nantes, France)

- Robotique et Entités Artificielles (1997)
- Links seven laboratories in France
- I was welcomed in Fall 1998 during a sabbatical in Strasbourg

Two Further Introductions…

RABBIT
(Grenoble, France)

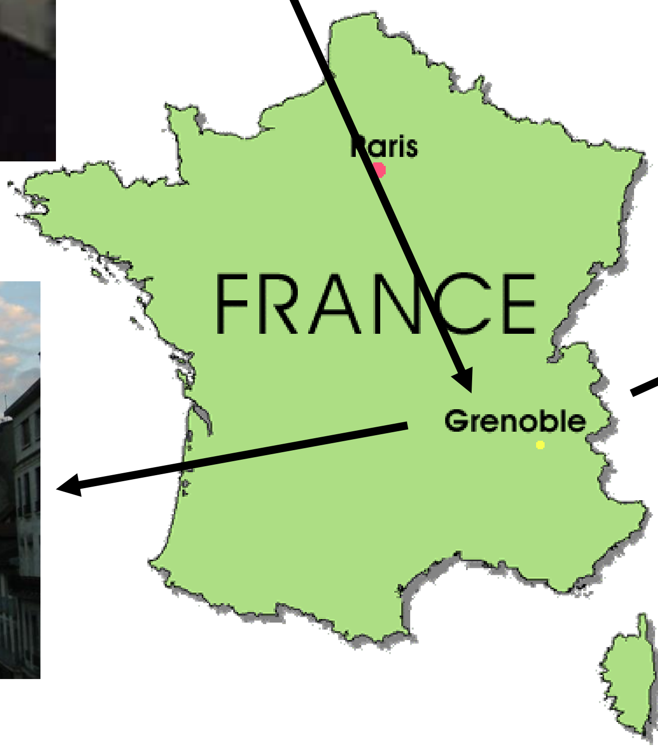


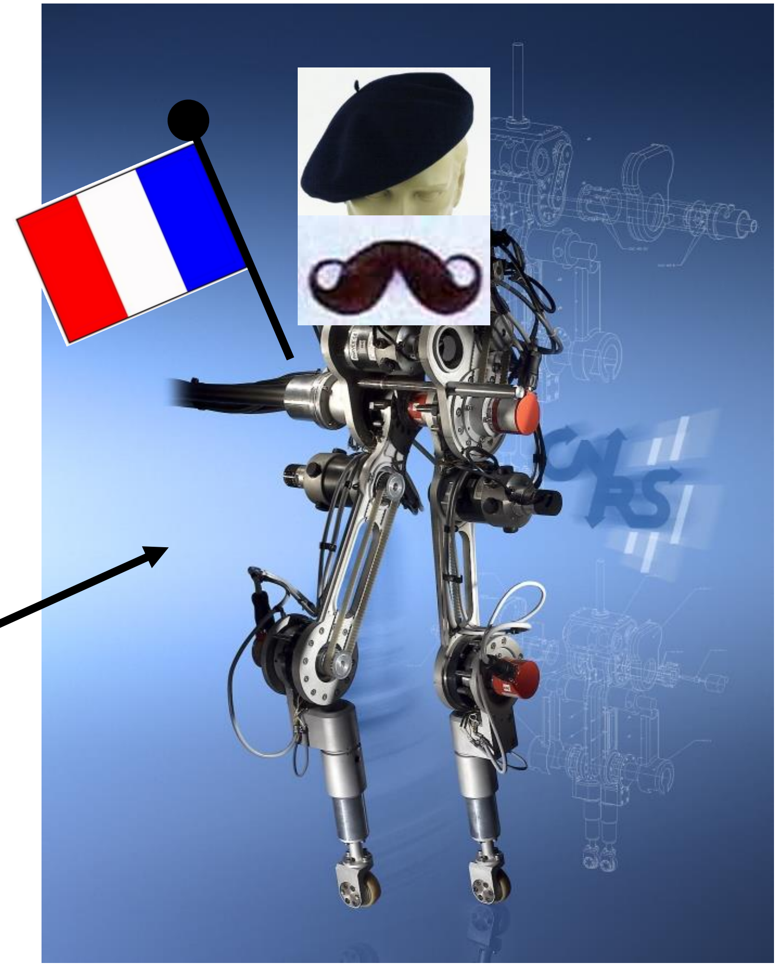
Eric Westervelt
(Ohio State Univ.)
(Asst. Professor)

Two Further Introductions…

Carlos Canudas-de-Wit

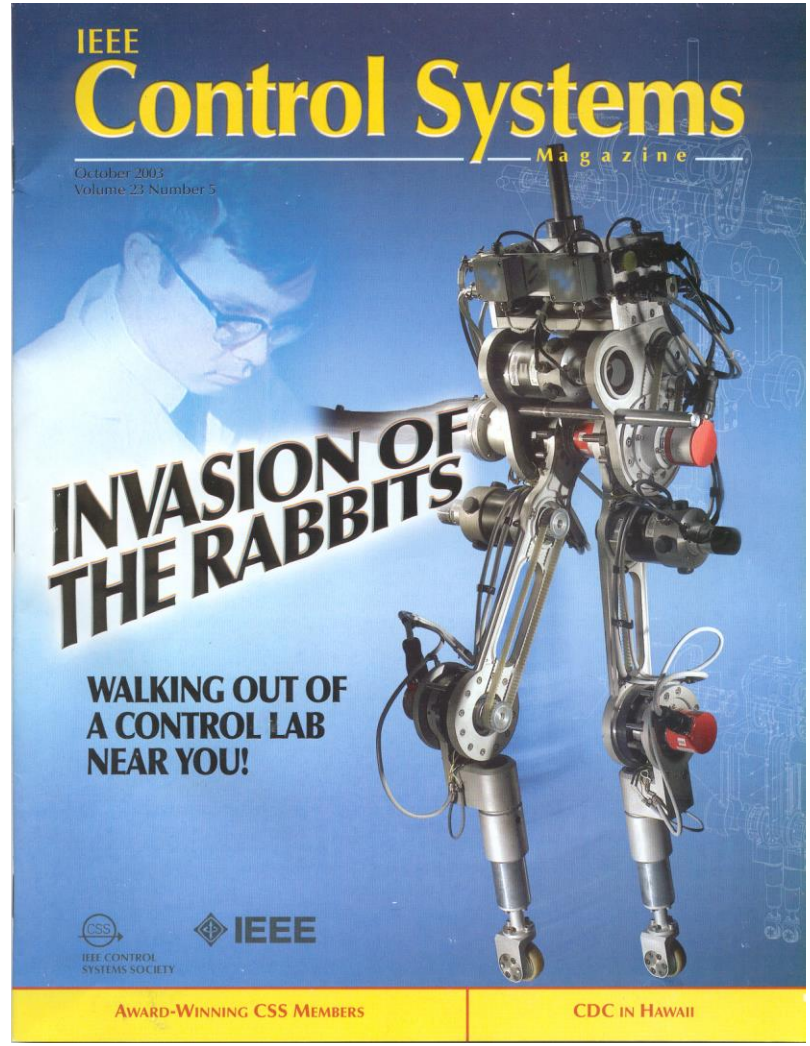
RABBIT
(Grenoble, France)





October 2003 Issue

- CSM paper is very conceptual
- Full details are in various IEEE-TAC & IJRR papers
- See my web site for listing of papers and many more videos (type ‘ grizzle’ into *Google*)

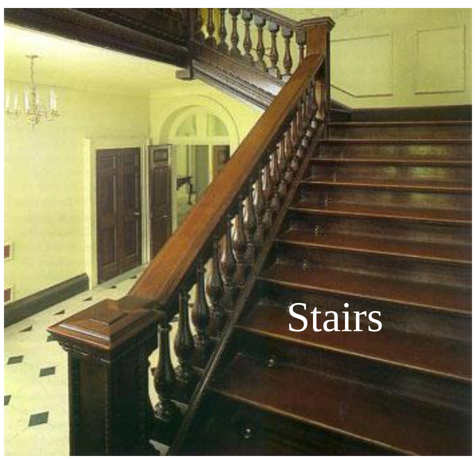


Outline

- Bipedal background
  - Why study mechanical bipedal walking?
  - What is known about stable gaits?
  - How to model a bipedal walking robot?
- A new look at feedback control for bipeds
  - Finding and exploiting problem structure
  - The key is a two dimensional (hybrid) dynamic
  - Feedback design with the Hybrid Zero Dynamics
- Experiments on RABBIT
  - Walking and running
- Conclusions

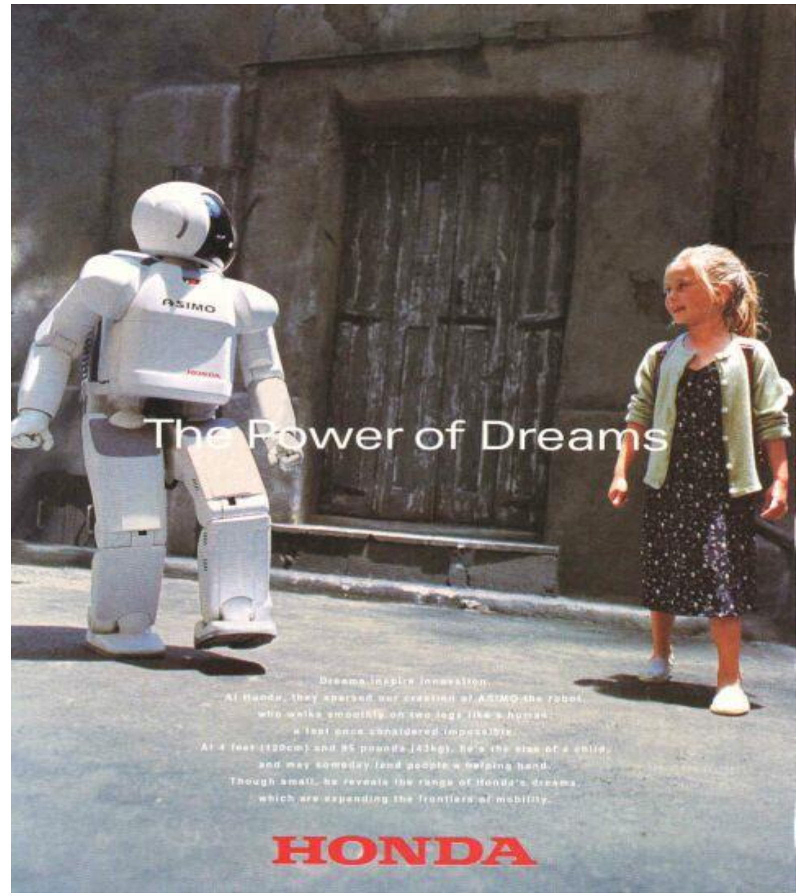
Why biped walking? (robotics)

Increased mobility…



Ladders

The fascination of anthropomorphic robots…



Why biped walking? (people)

Prosthetics: Leg Design

[**Ottobock** C-Leg]

Rehabilitation of Walking

- Strokes
- Spinal Injury

(Weight suspended treadmill therapy)

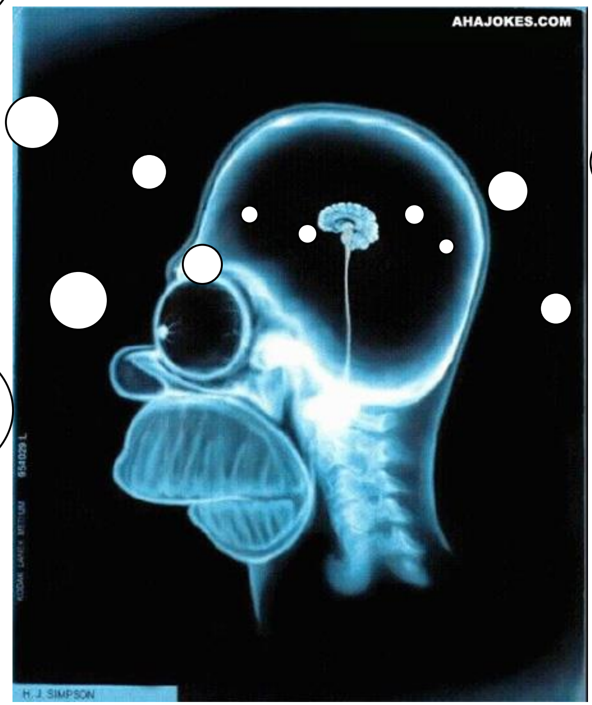
AutoAmbulator

Lokomat (Morari et al.)

Why biped walking? (control)

Intellectual Curiosity

Awesome Experiments



Way Cool Mathematics

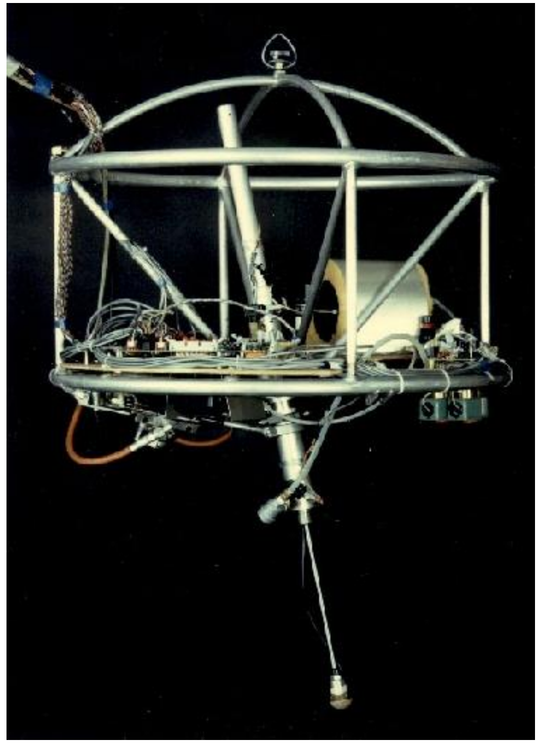
Low-Hanging Fruit

Two Approaches to Locomotion and Control

- Analytical Methods
  - rigorous model-based analysis
  - success with very little tweaking
  - experimentation is used to test theory
- Heuristic Methods
  - based on intuition
  - trial and error – many trials before success
  - uncertainty as to why success or failure was the outcome
  - usually produces awkward motions--slow, crouching gaits

Analytical Approaches to Locomotion and Control

- **One-legged Hopper**
  - **Koditschek & Beuhler 1991**
  - **Francois & Samson 1998**



Raibert (1984)

Analytical Approaches to Locomotion and Control

- **One-legged Hopper**
  - **Koditschek & Beuhler 1991**
  - **Francois & Samson 1998**

Raibert (1984)

Analytical Approaches to Locomotion and Control

- One-legged Hopper
- **Passive Robots**
  - **McGeer 1990**
  - **Espiau & Goswani 1994**
  - **Ruina et al. 1997-2004**
  - **Howell & Baillieul 1998**
  - **Kuo et al. 1999-2004**

3-D

Collins and Ruina (2000)

Gravity Powered Walking Down a Gentle Slope…The Ultimate in Efficiency!

Analytical Approaches to Locomotion and Control

- One-legged Hopper
- **Passive Robots**
  - **McGeer 1990**
  - **Espiau & Goswani 1994**
  - **Ruina et al. 1997-2004**
  - **Howell & Baillieul 1998**
  - **Kuo et al. 1999-2004**

3-D

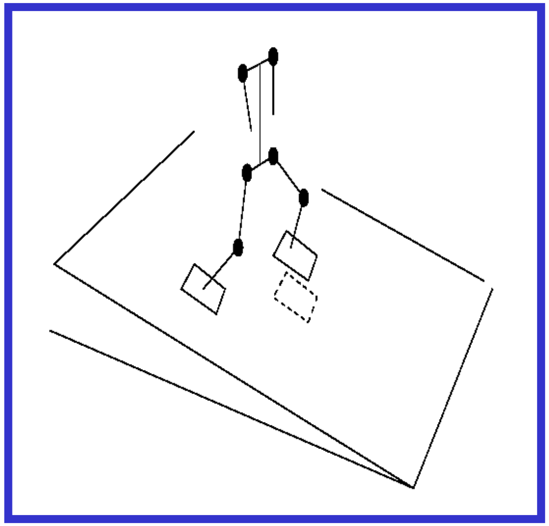
Collins and Ruina (2000)

Gravity Powered Walking Down a Gentle Slope…The Ultimate in Efficiency!

Analytical Approaches to Locomotion and Control

- One-legged Hopper
- Passive Robots
- **Lifting Passive Gaits to Fully-Actuated Bipeds**
  - **Spong 1997**
  - **Spong & Bullo 2002**

(Please build me!)

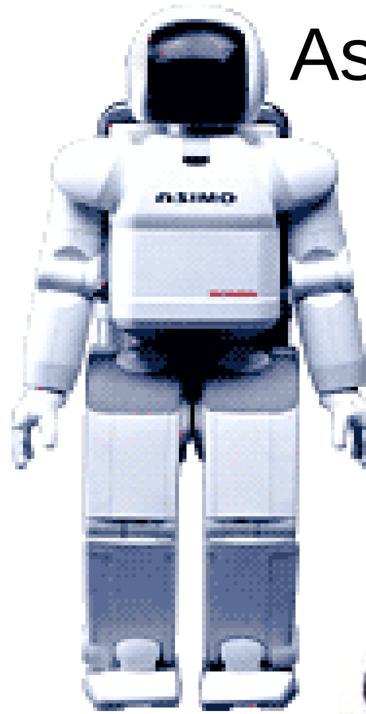


Powered walking on **flat** and **sloped** surfaces!

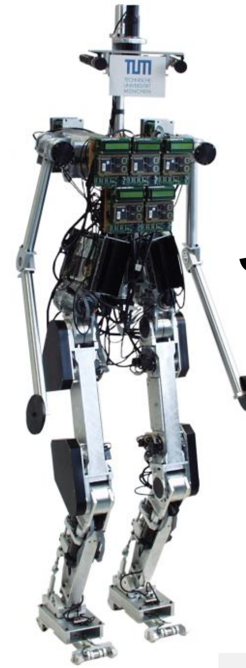
Spong & Bullo (2002)

Most Powered Biped Robots use <u>Heuristics</u> for Controller Design

- ZMP (Zero Moment Point)
  - Asimo [Honda ’96 →],
    >$150,000,000 [dev. cost] and
    $1,000,000 per robot
  - QRIO [Sony, 2001]
- Intuition
  - Spring Flamingo
    [MIT Leg Lab ’96-’00]
- Other Approx. Notions
  - Many

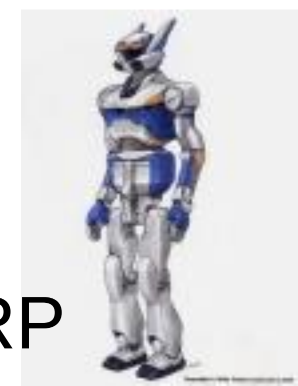


Asimo



Johnnie

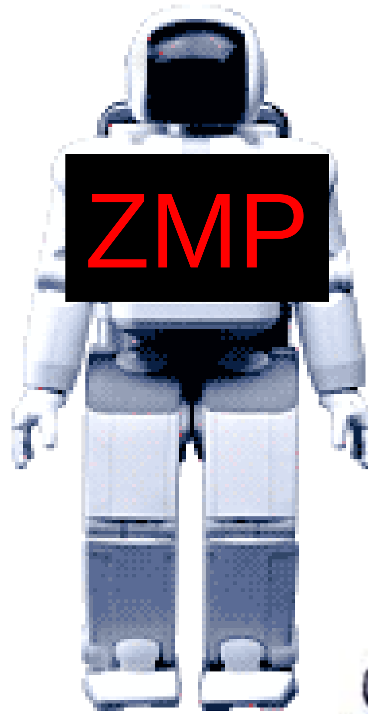
QRIO

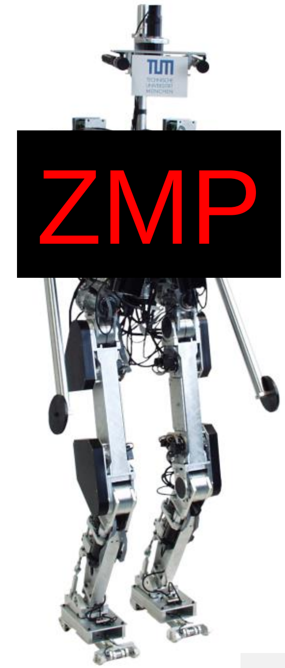


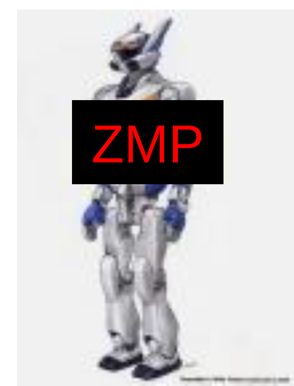
HRP

Most Powered Biped Robots use Heuristics for Controller Design

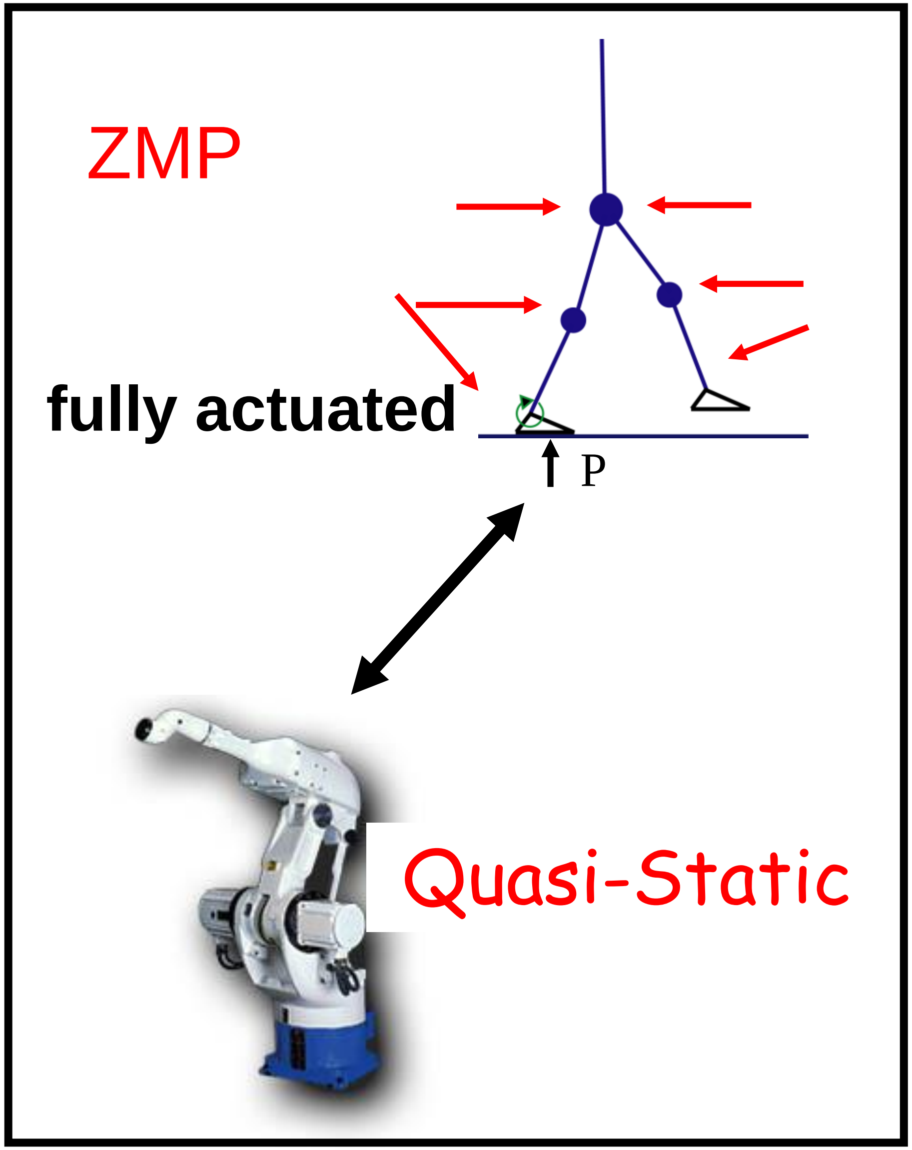
- ZMP (Zero Moment Point)
  - Asimo [Honda '96 →], >$150,000,000 [dev. cost] and $1,000,000 per robot
  - QRIO [Sony, 2001]
- Intuition
  - Spring Flamingo [MIT Leg Lab '96-'00]
- Other Approx. Notions
  - Many

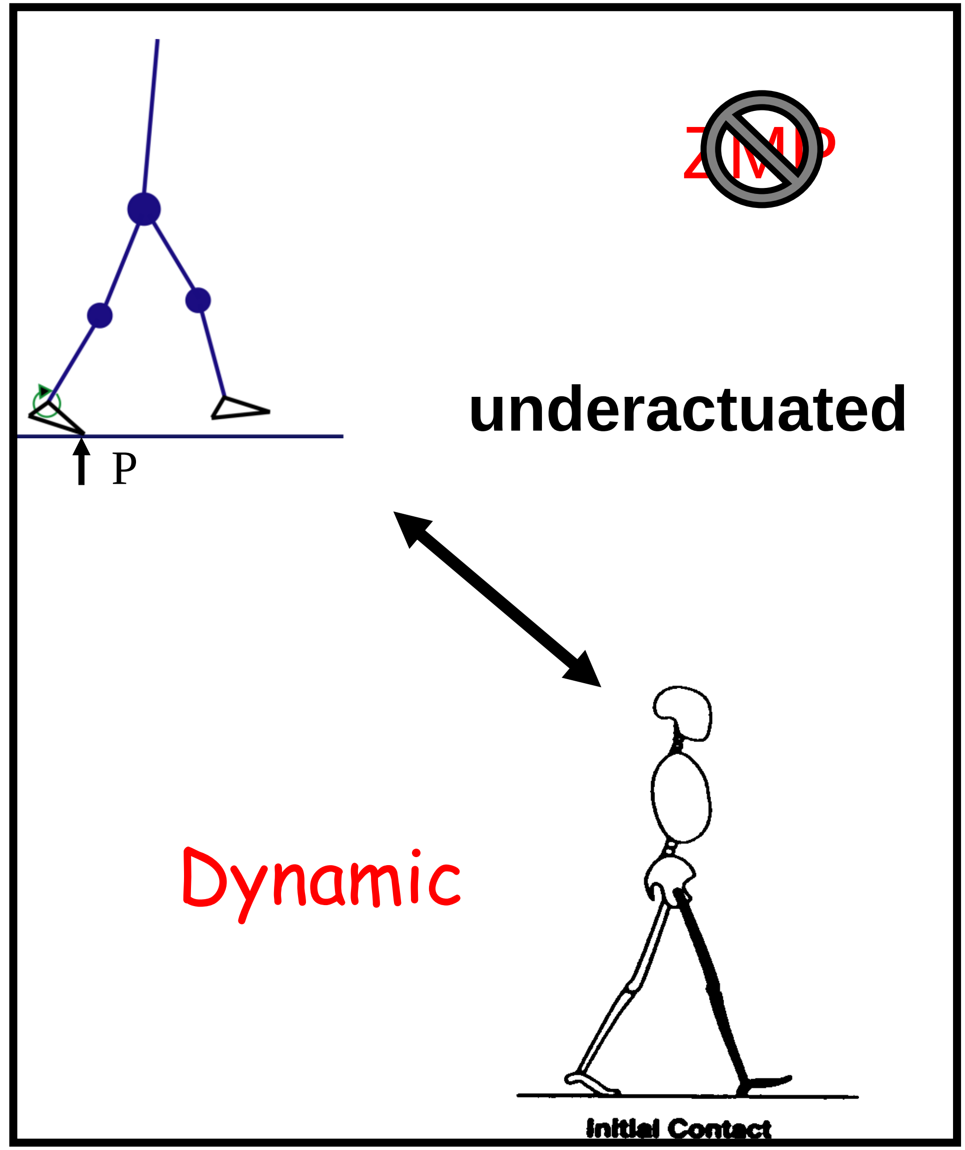




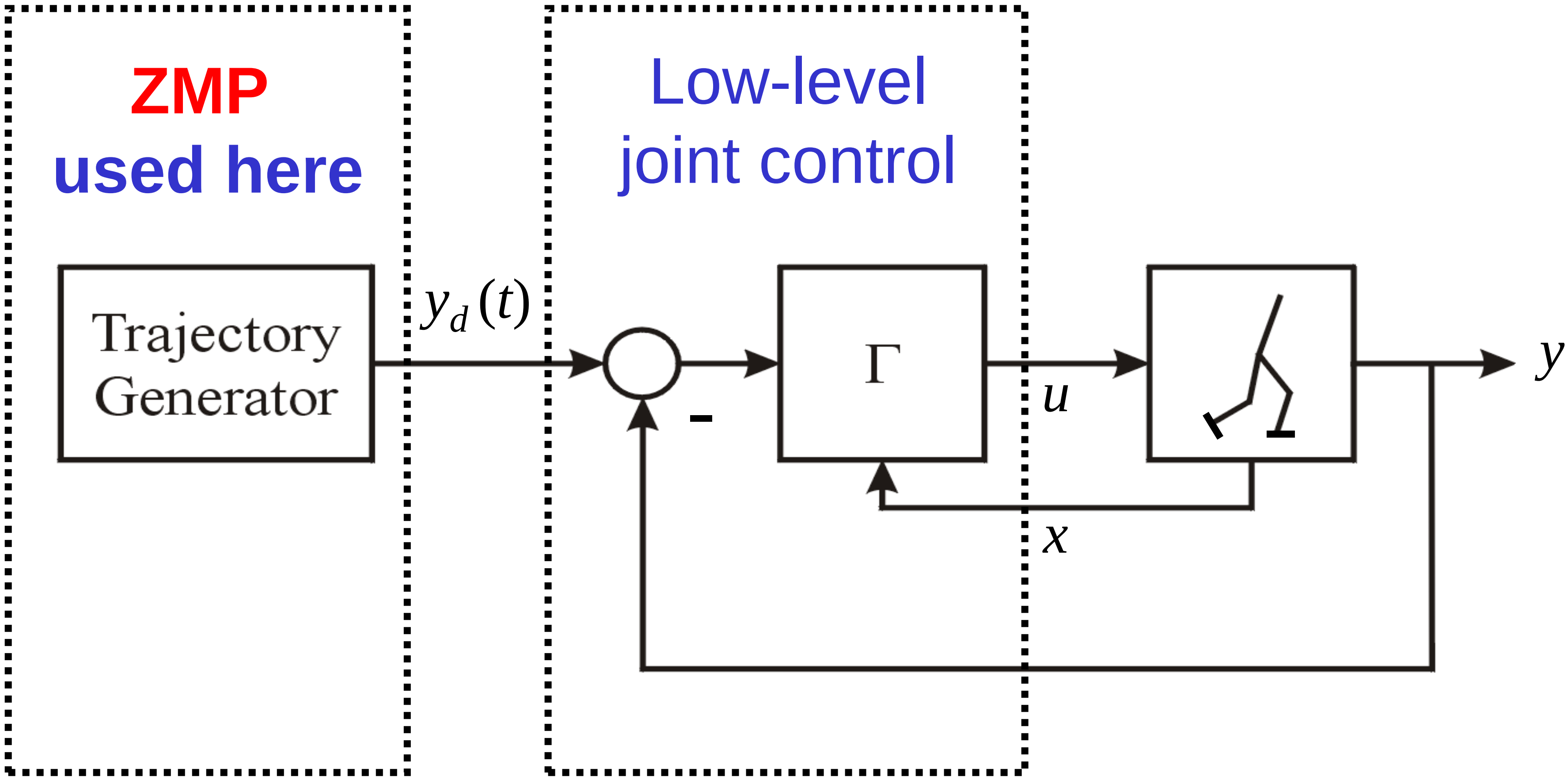


ZMP = Flat-Footed Walking





Prevailing Control Approach is ZMP-Based Trajectory Tracking



Prevailing Approach Fights Natural Dynamics of Walking

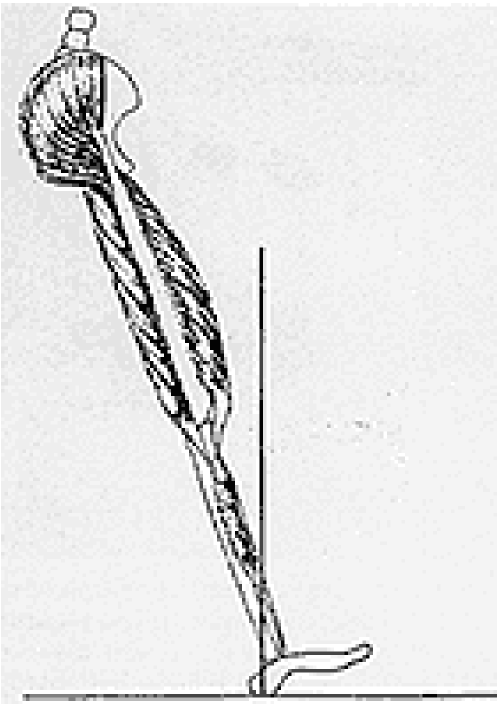
- Heuristic Methods
  - based on intuition
  - trial and error – many trials before success
  - uncertainty as to why success or failure was the outcome
  - usually produces awkward motions--**slow, crouching gaits**

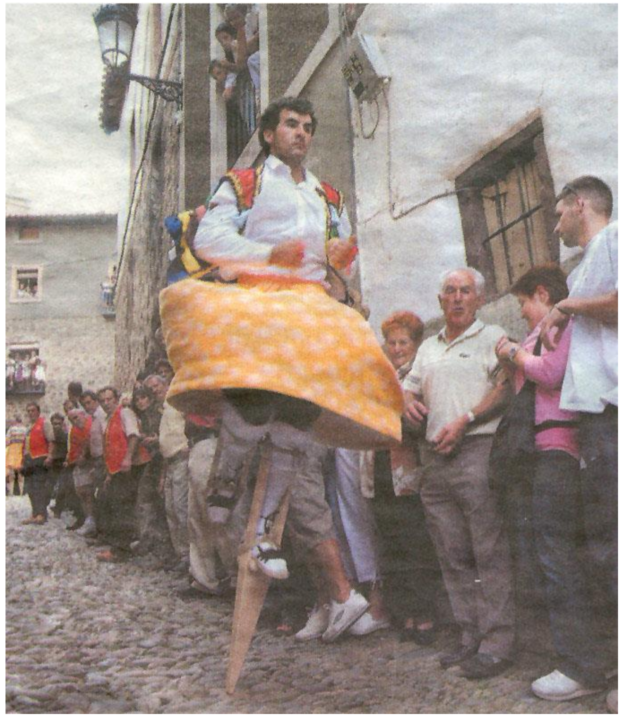
Walking

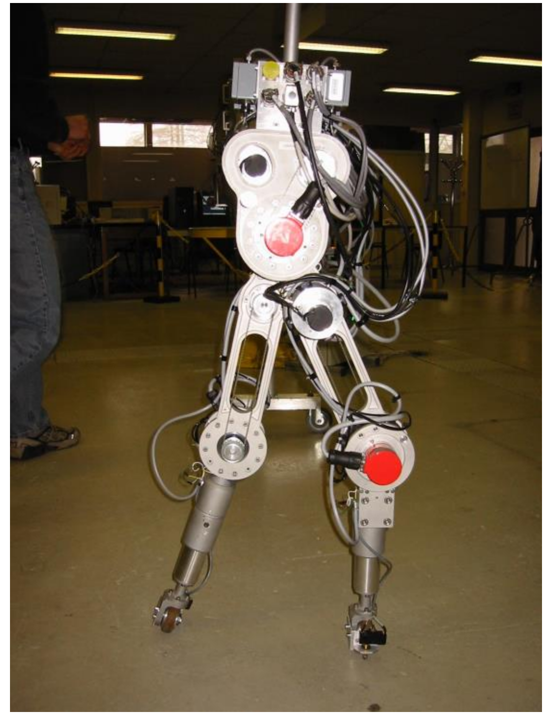
Running!

Qrio-Sony

Prevailing Approach Fights Natural Dynamics of Walking



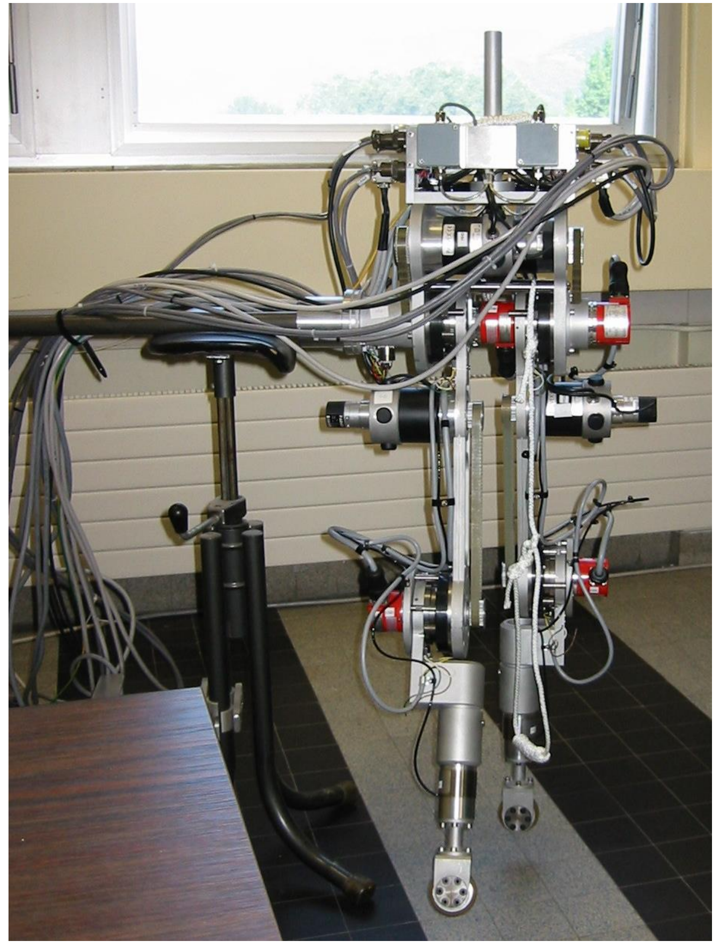




RABBIT obliges you to EXPLOIT dynamics of walking.

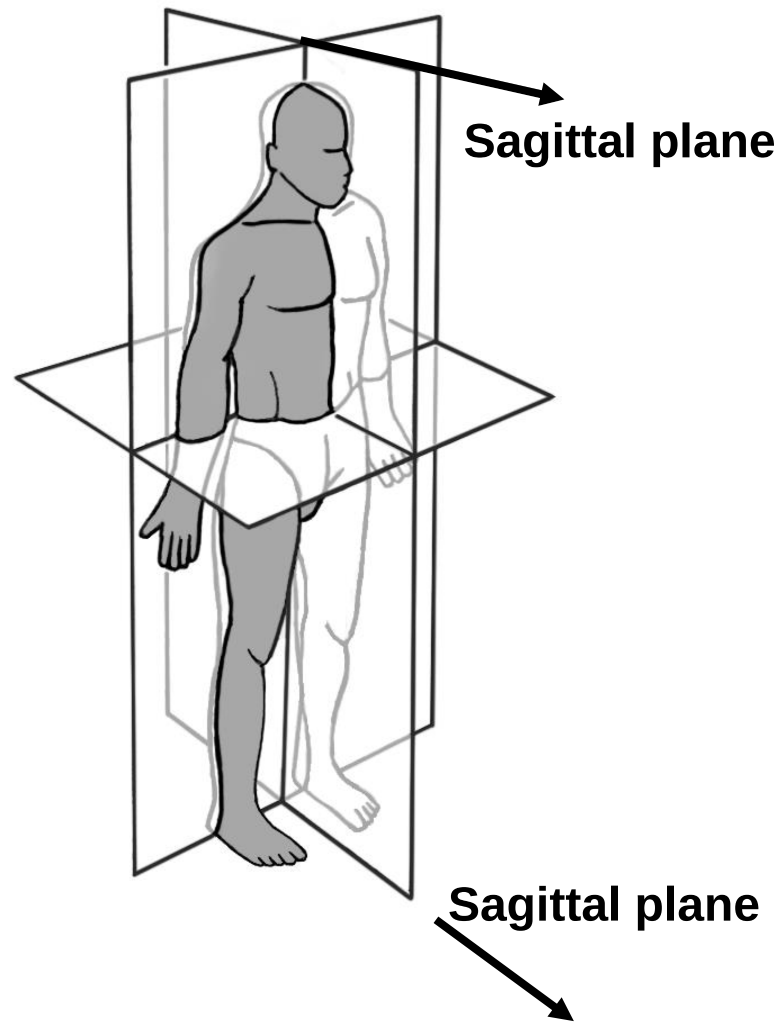
RABBIT: Simplest Mechanism Capable of Quasi-Anthropomorphic Gait

- Two legs, knees, a torso



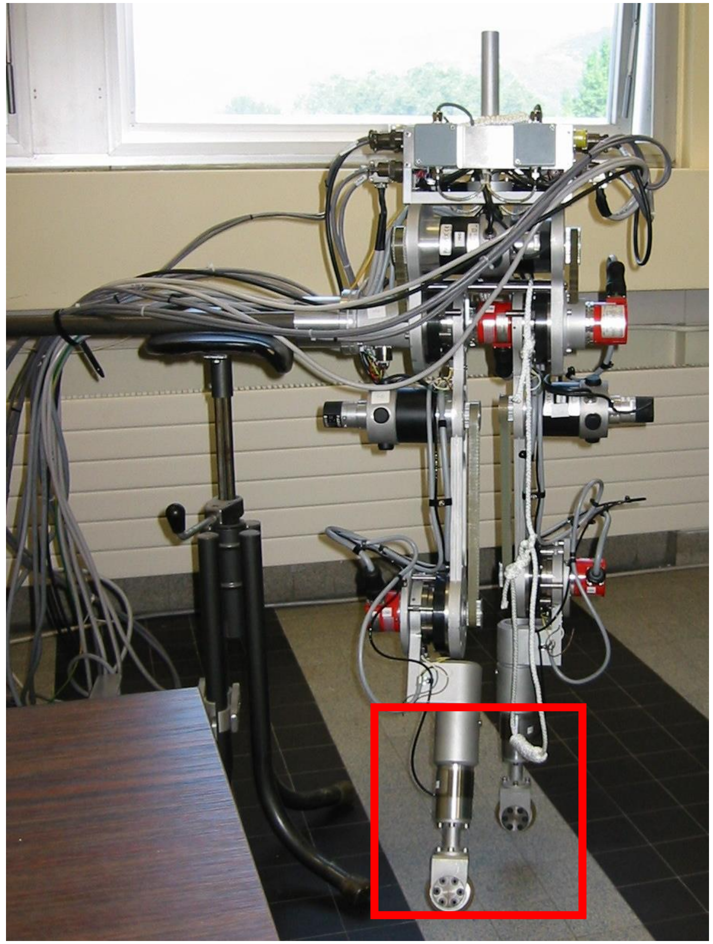
RABBIT: Simplest Mechanism Capable of Quasi-Anthropomorphic Gait

- Two legs, knees, a torso
- Sagittal plane dynamics
- Side-to-side stability assured by a bar



RABBIT: Simplest Mechanism Capable of Quasi-Anthropomorphic Gait

- Two legs, knees, a torso
- Sagittal plane dynamics
- Side-to-side stability assured by a bar
- Point feet = No ZMP = Need new control theory!



RABBIT: Simplest Mechanism Capable of Quasi-Anthropomorphic Gait

- Two legs, knees, a torso
- Sagittal plane dynamics
- Side-to-side stability assured by a bar
- Point feet = No ZMP!

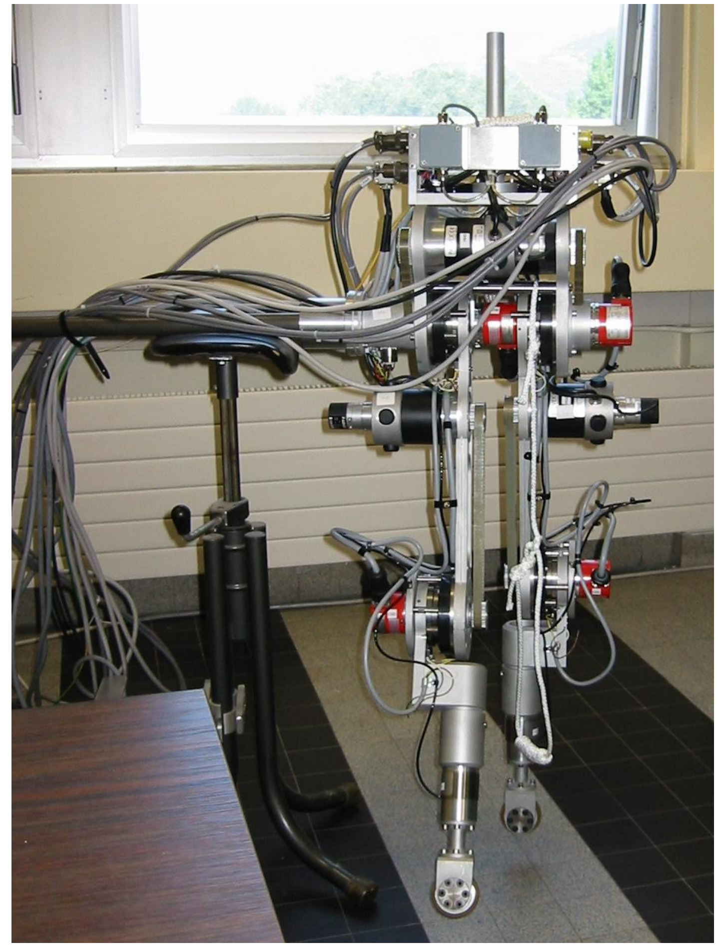
Typical Gait

LAG: Laboratoire Automatique de Grenoble

RABBIT: Simplest Mechanism Capable of Quasi-Anthropomorphic Gait

Question everyone asks:

Does the bar hold up the robot?

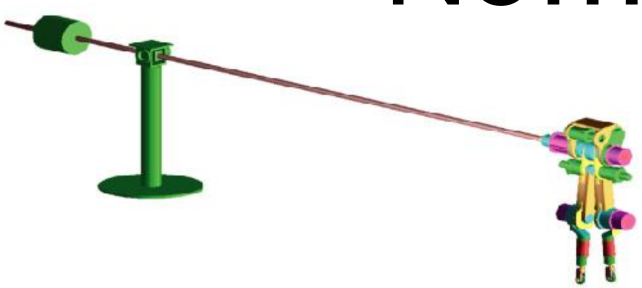


RABBIT: Simplest Mechanism Capable of Quasi-Anthropomorphic Gait

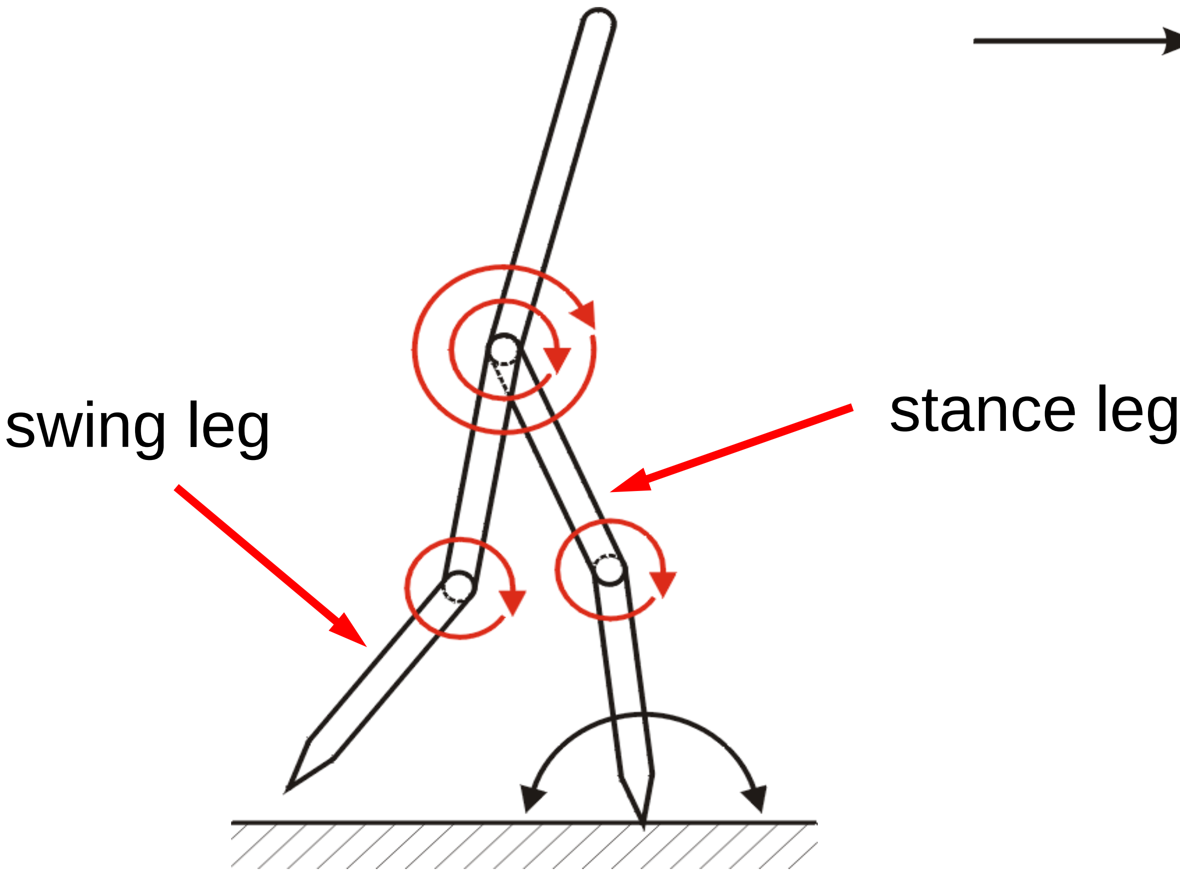
Question everyone asks:

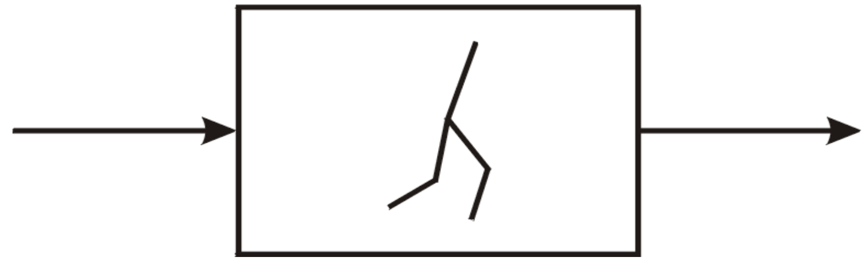
Does the bar hold up the robot?

No….



RABBIT is Planar & Underactuated

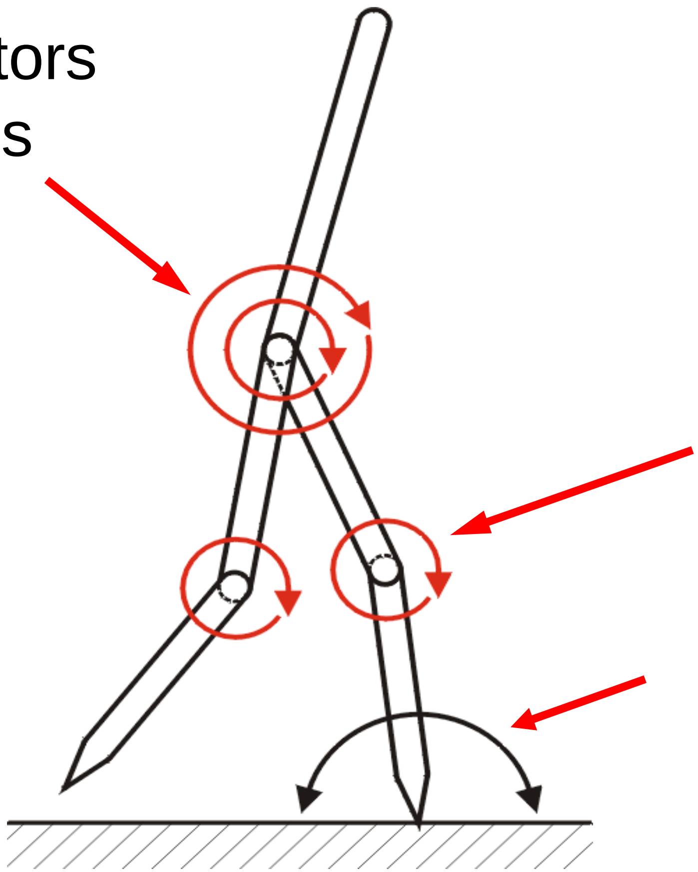




32 kg mass and 1.425 m tall

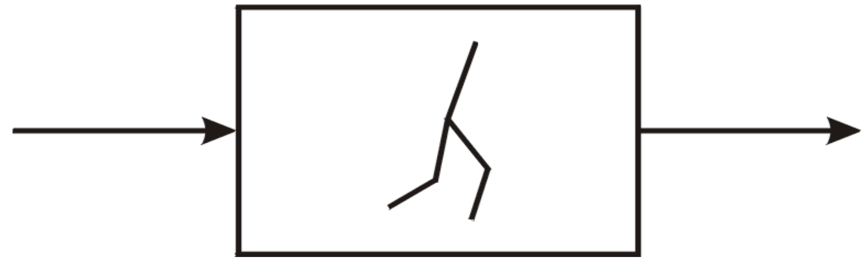
RABBIT is Planar & Underactuated

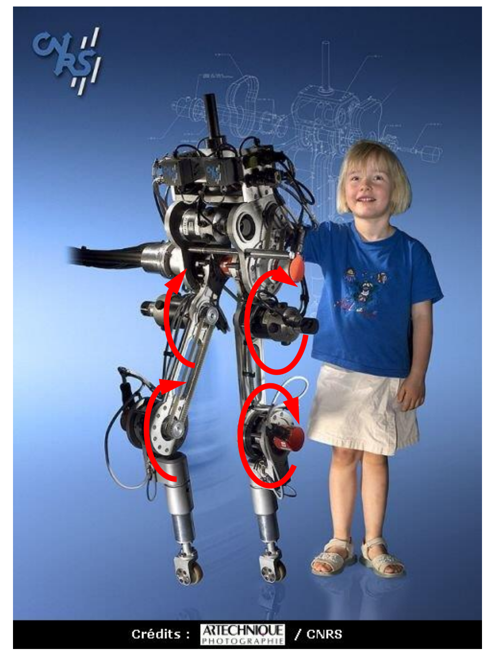
2 actuators at hips



1 actuator each knee

unactuated DOF

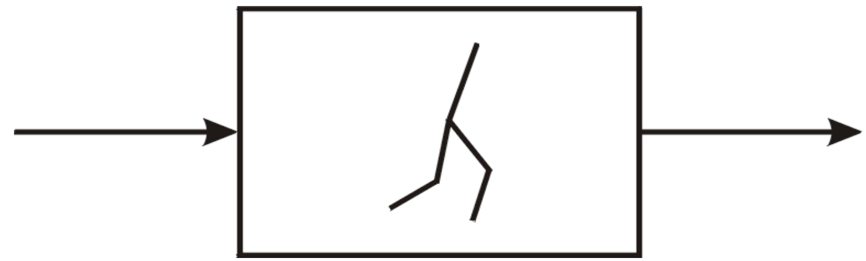




32 kg mass and 1.425 m tall

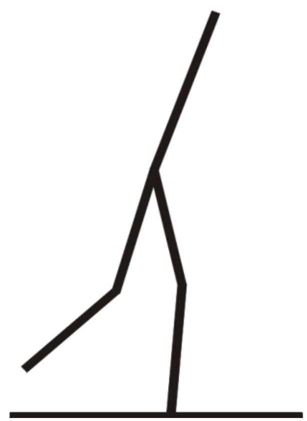
Robot Model: *SS* + *DS* = Hybrid

Normal walking:

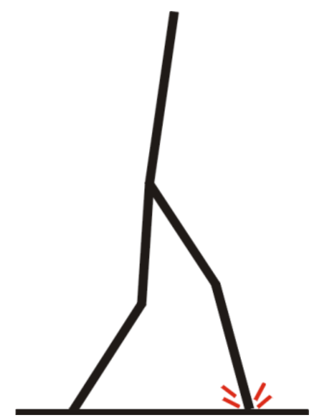


… *SS*, *DS*, *SS*, *DS*, …

SS — Single Support

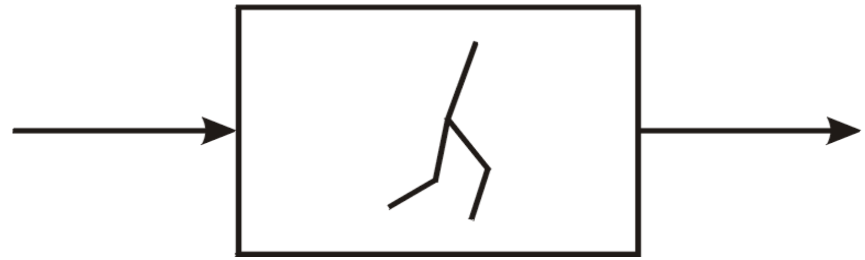


DS — Double Support



Robot Model: *SS* + *DS* = Hybrid

Normal walking:

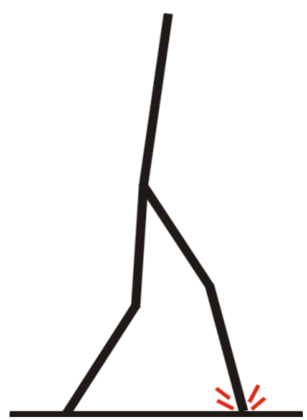


… *SS*, *DS*, *SS*, *DS*, …

SS — Single Support

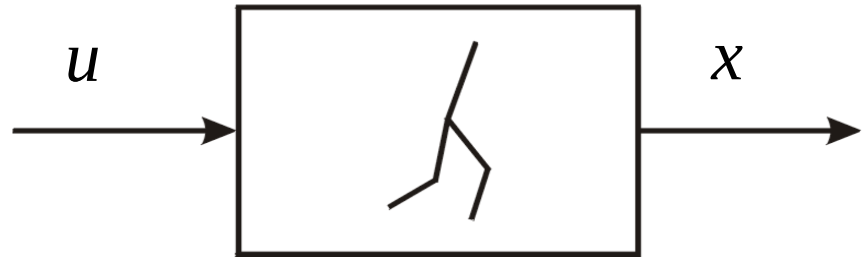
DS — Double Support

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Bu$$



Robot Model: *SS* + *DS* = Hybrid

Normal walking:



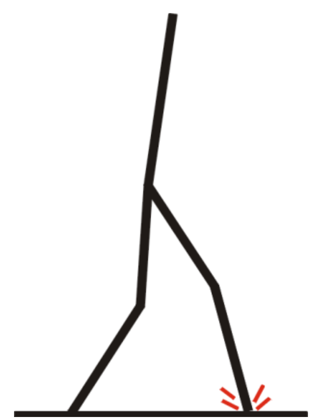
… *SS*, *DS*, *SS*, *DS*, …

SS — Single Support

DS — Double Support

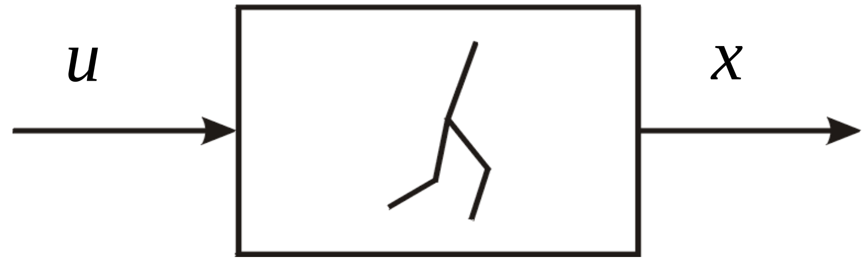
$$\dot{x} = f(x) + g(x)u$$

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$



Robot Model: *SS* + *DS* = Hybrid

Normal walking:



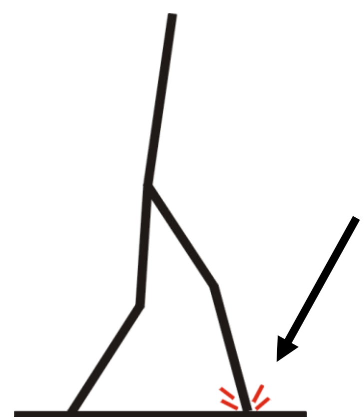
… *SS*, *DS*, *SS*, *DS*, …

SS — Single Support

$$\dot{x} = f(x) + g(x)u$$

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

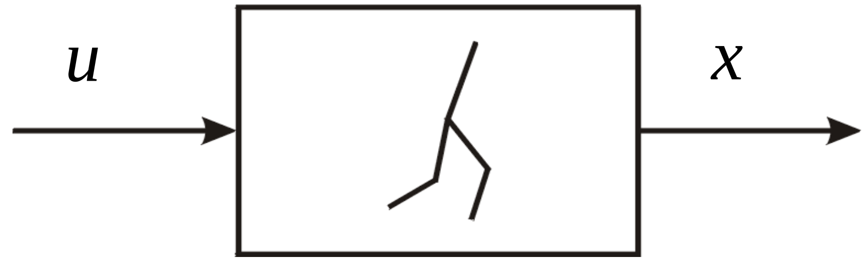
DS — Double Support



Impulsive Force!

Robot Model: *SS* + *DS* = Hybrid

Normal walking:



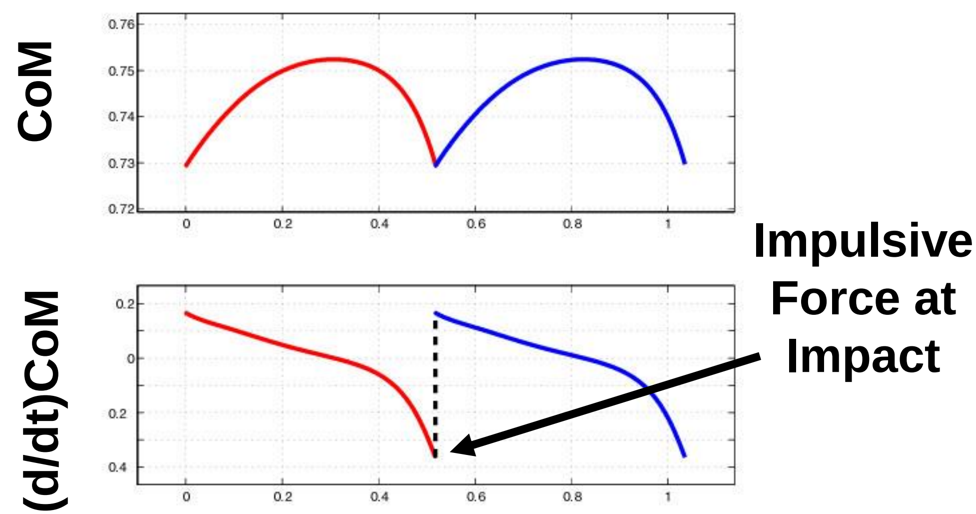
… *SS*, *DS*, *SS*, *DS*, …

SS — Single Support

$$\dot{x} = f(x) + g(x)u$$

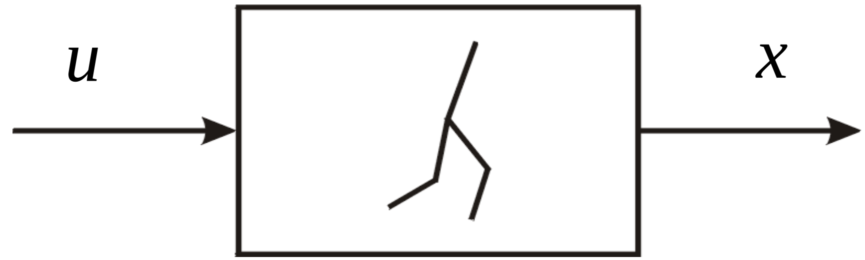
$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

DS — Double Support



Robot Model: *SS* + *DS* = Hybrid

Normal walking:



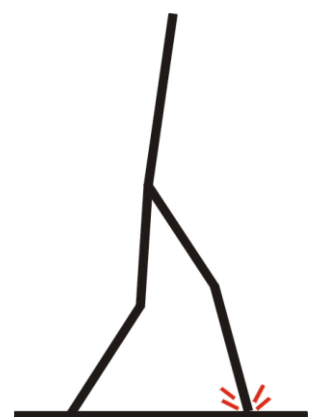
… *SS*, *DS*, *SS*, *DS*, …

SS — Single Support

$$\dot{x} = f(x) + g(x)u$$

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

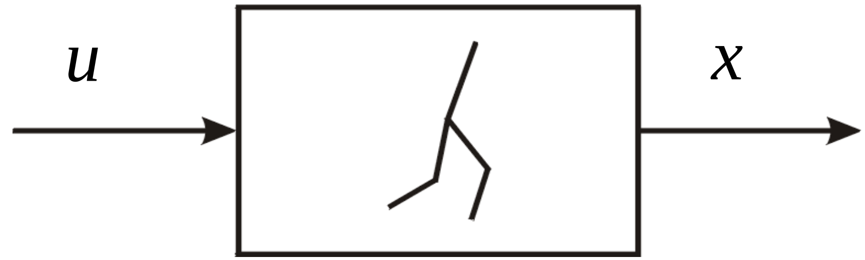
DS — Double Support



$$x^{+} = \Delta(x^{-})$$

Robot Model: *SS* + *DS* = Hybrid

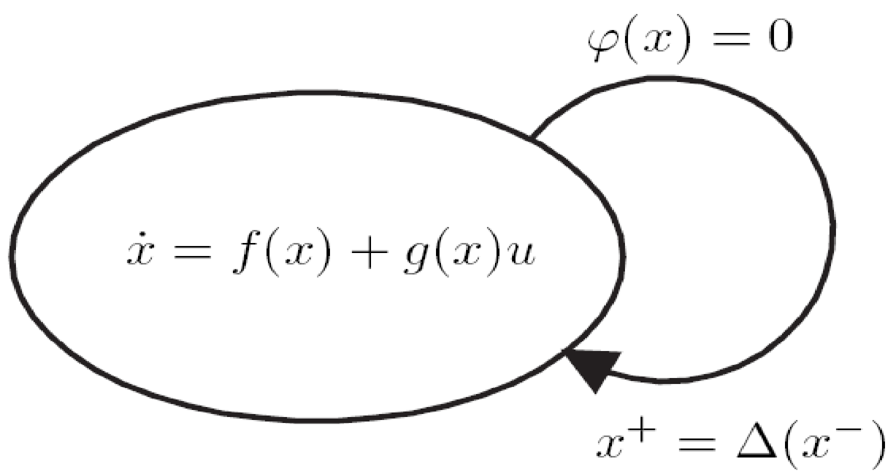
Normal walking:



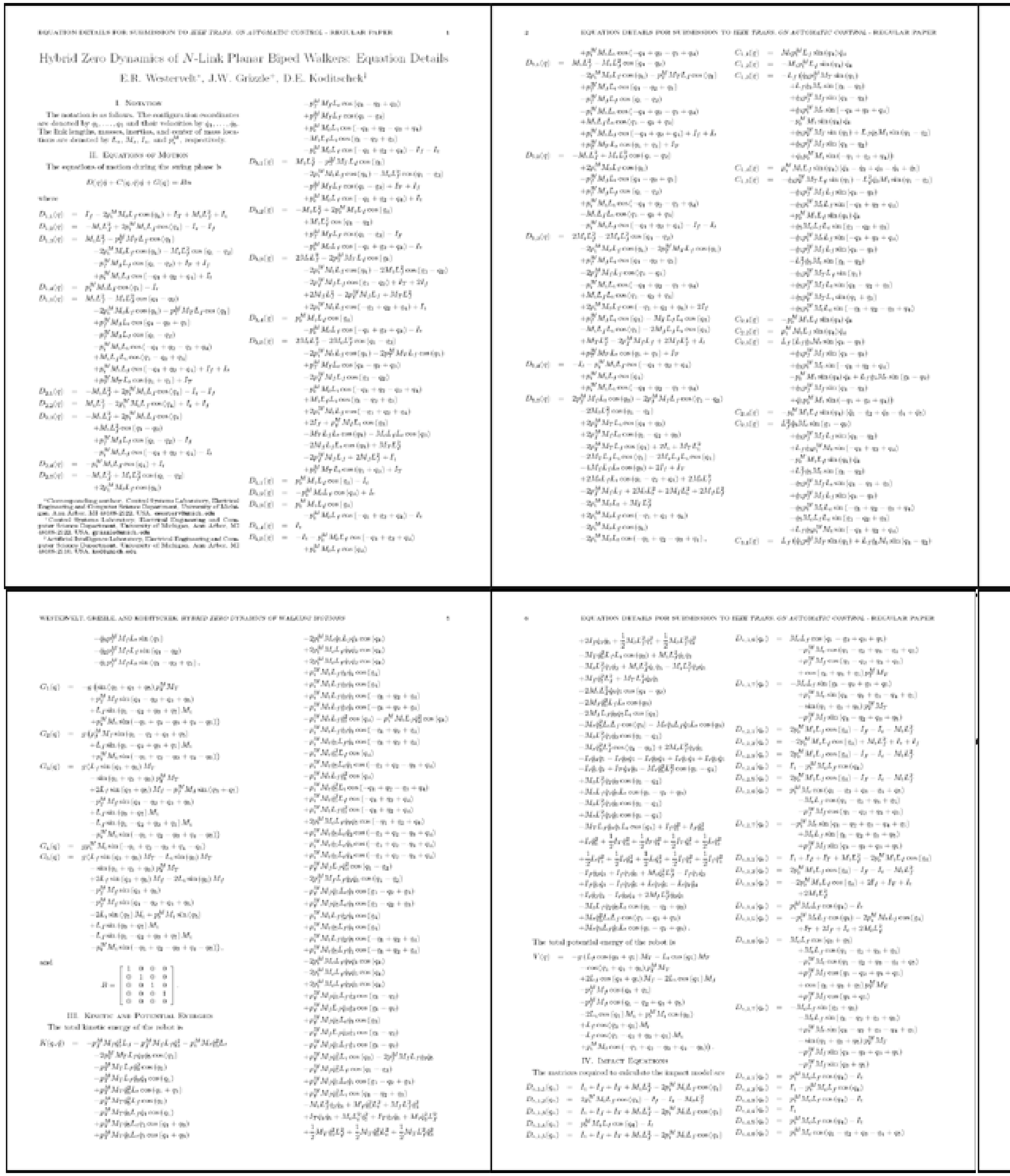
… *SS*, *DS*, *SS*, *DS*, …

SS — Single Support *DS* — Double Support

10 differential
equations
&
impact map



Terms in the Model …Oh my!



$$D_{e,4,7}(q_e) = -p_t^M M_t \sin(q_1 - q_2 + q_3 - q_4 + q_5)$$

$$D_{e,5,1}(q_e) = I_t + I_f + I_T + M_t L_f^2 - 2p_t^M M_t L_f \cos(q_4)$$

$$D_{e,5,2}(q_e) = 2p_t^M M_t L_f \cos(q_4) - I_f - I_t - M_t L_f^2$$

$$D_{e,5,3}(q_e) = -p_t^M M_t L_f \cos(q_3) - 2p_t^M M_t L_f \cos(q_4) + I_T + 2I_f + I_t + 2M_t L_f^2$$

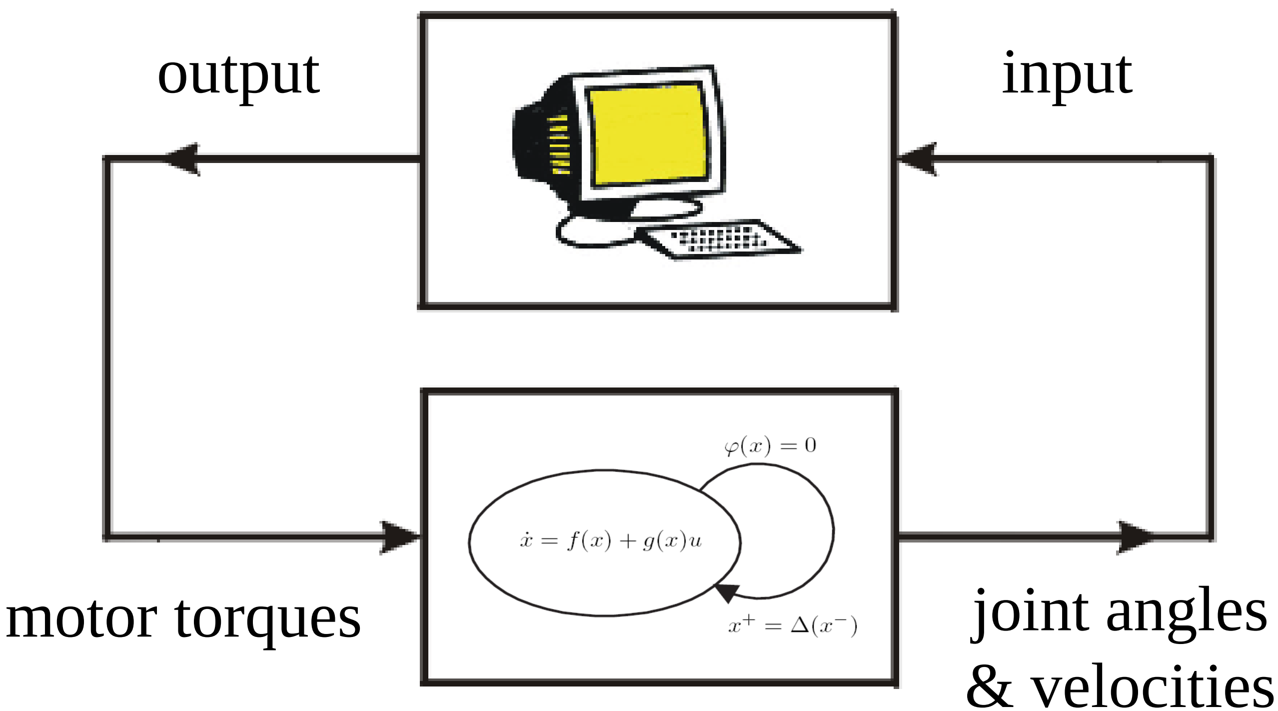
$$D_{e,5,4}(q_e) = p_t^M M_t L_f \cos(q_4) - I_t$$

$$D_{e,5,5}(q_e) = -2p_t^M M_t L_f \cos(q_3) - 2p_t^M M_t L_f \cos(q_4) + I_T + 2I_f + 2I_t + 2M_t L_f^2$$

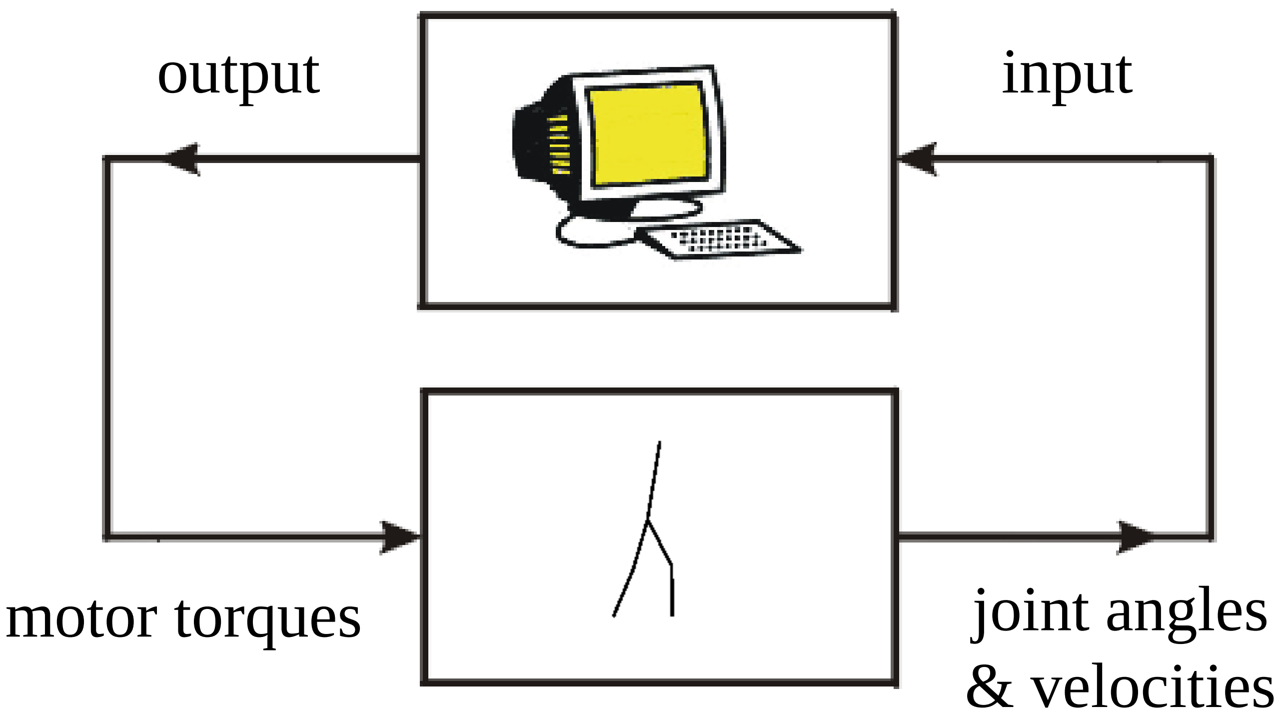
$$\begin{aligned} D_{e,5,6}(q_e) = {} & M_t L_f \cos(q_3 + q_5) \\ & + M_t L_f \cos(q_1 - q_2 + q_3 + q_5) \\ & - p_t^M M_t \cos(q_1 - q_2 + q_3 - q_4 + q_5) \\ & + p_f^M M_f \cos(q_1 - q_2 + q_3 + q_5) \\ & + \cos(q_1 + q_3 + q_5) p_T^M M_T \\ & + p_f^M M_f \cos(q_3 + q_5) - p_t^M M_t \cos(q_5) \end{aligned}$$

$$\begin{aligned} D_{e,5,7}(q_e) = {} & -M_t L_f \sin(q_3 + q_5) \\ & - M_t L_f \sin(q_1 - q_2 + q_3 + q_5) \\ & + p_t^M M_t \sin(q_1 - q_2 + q_3 - q_4 + q_5) \\ & - \sin(q_1 + q_3 + q_5) p_T^M M_T \\ & - p_f^M M_f \sin(q_1 - q_2 + q_3 + q_5) \\ & - p_f^M M_f \sin(q_3 + q_5) + p_t^M M_t \sin(q_5) \end{aligned}$$

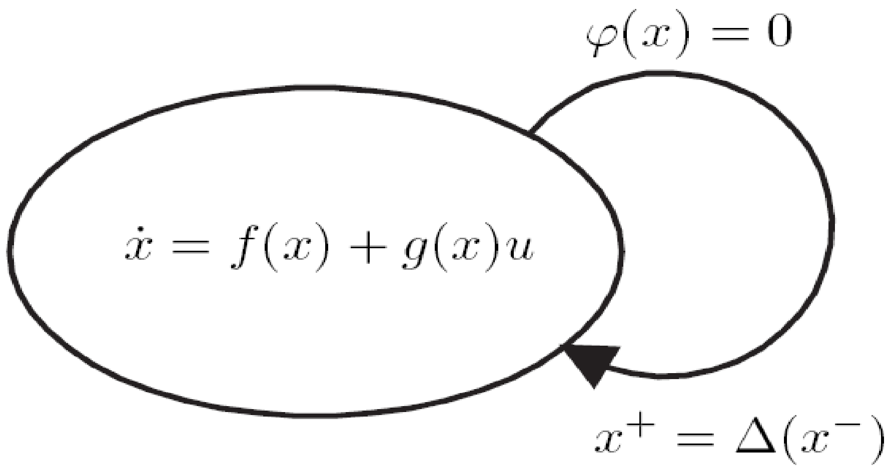
Aim: Design A Stable Periodic Orbit in a High-DOF Hybrid Model

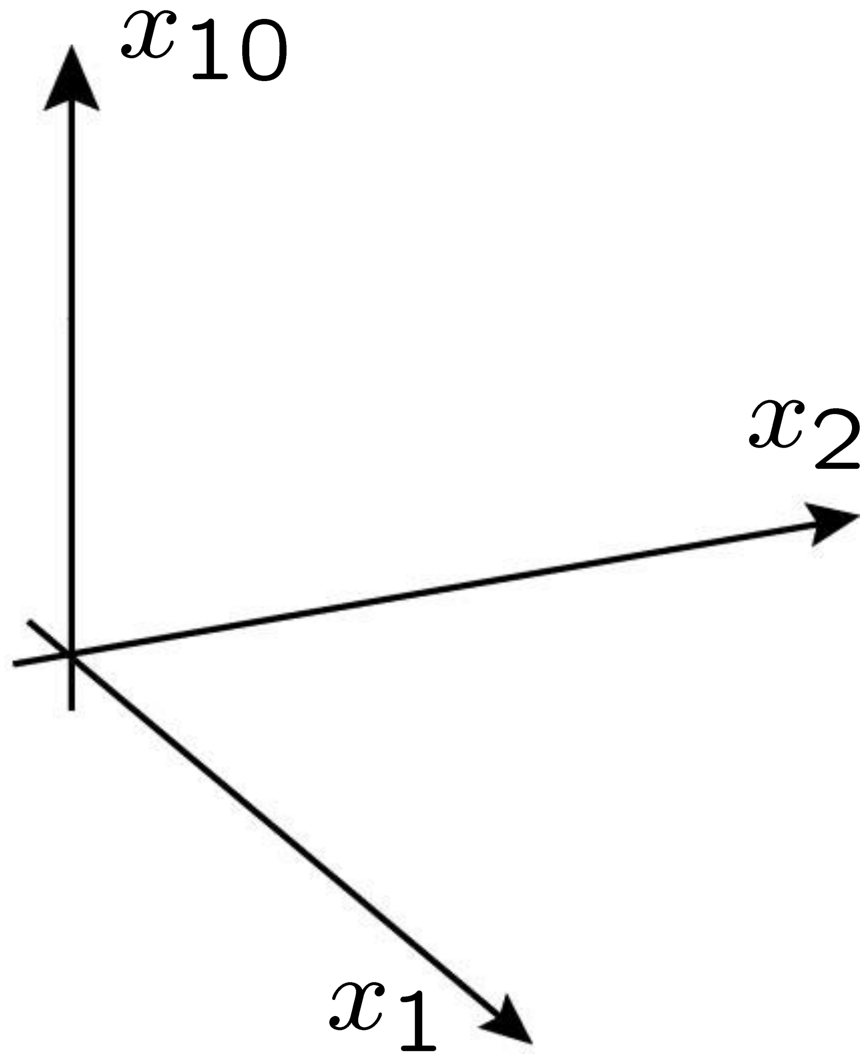


Aim: Design A Stable Periodic Orbit in a High-DOF Hybrid Model

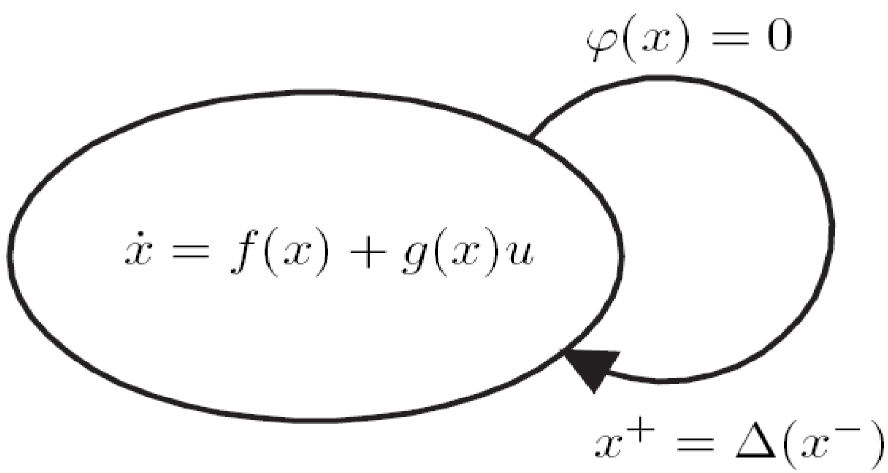


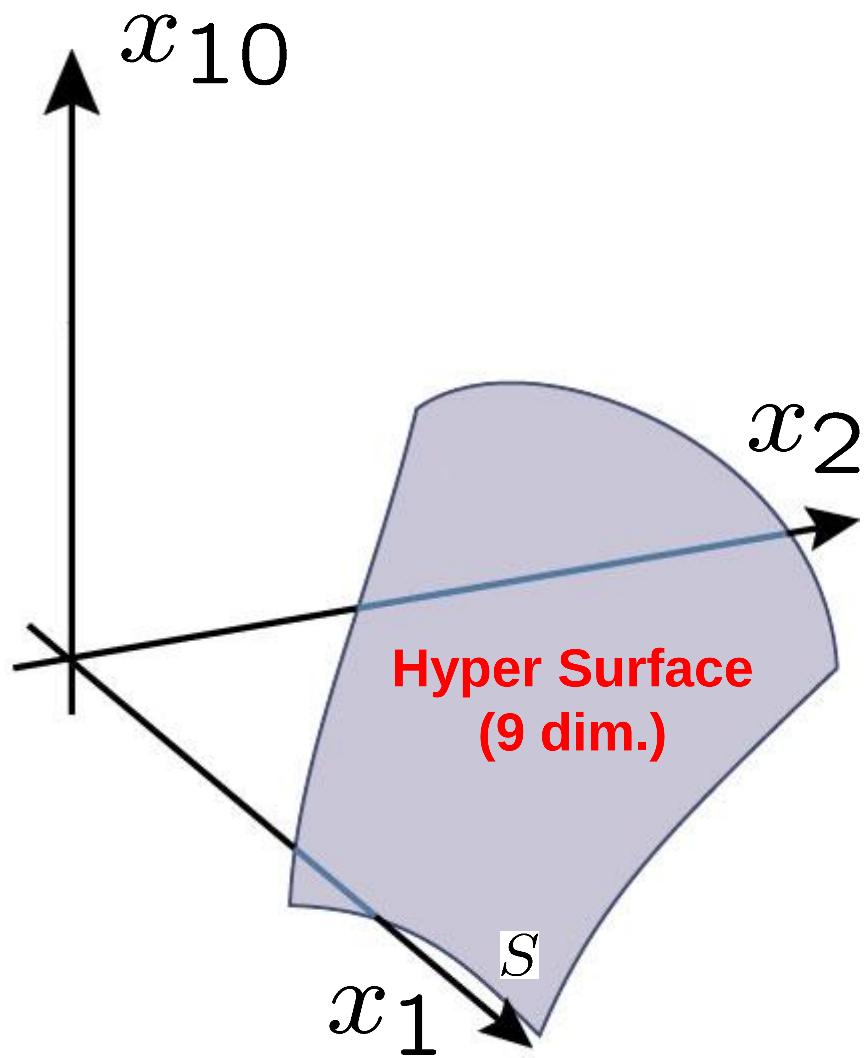
Aim: Design A Stable Periodic Orbit in a High-DOF Hybrid Model





Aim: Design A Stable Periodic Orbit in a High-DOF Hybrid Model

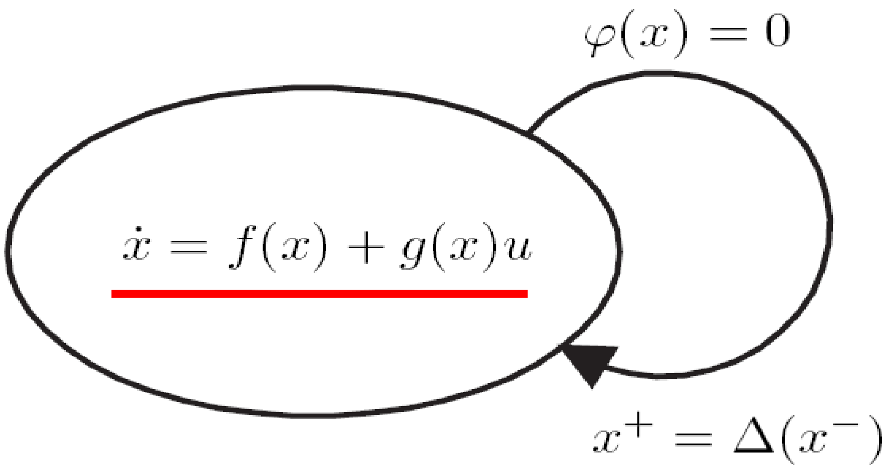


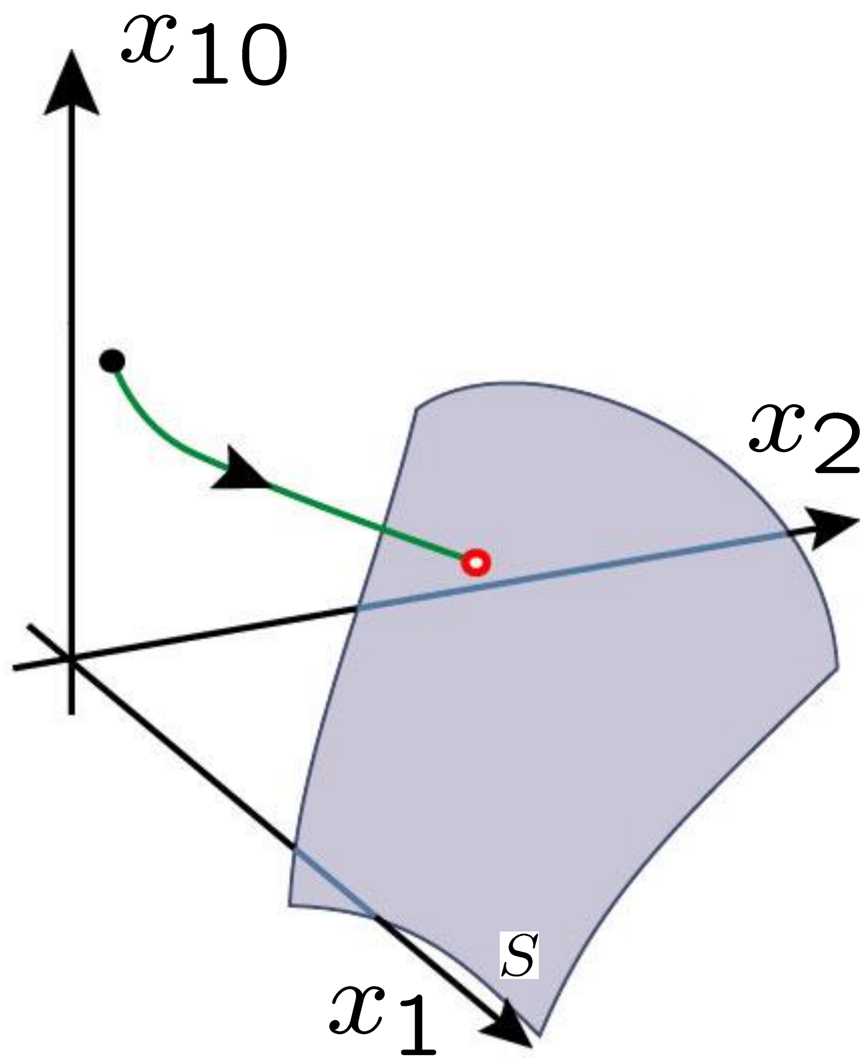


Switching Surface or Impact Surface

$$S = \{x \in \mathcal{X} \mid \varphi(x) = 0\}$$

Aim: Design A Stable Periodic Orbit in a High-DOF Hybrid Model



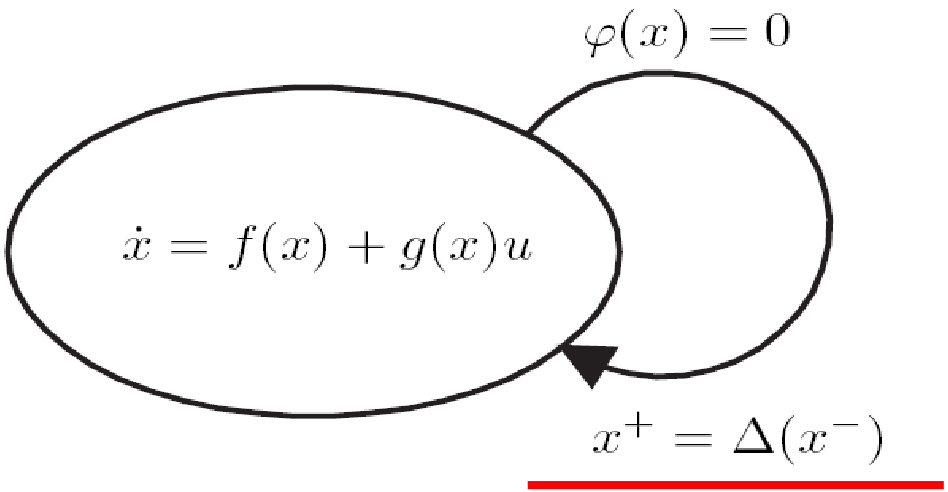


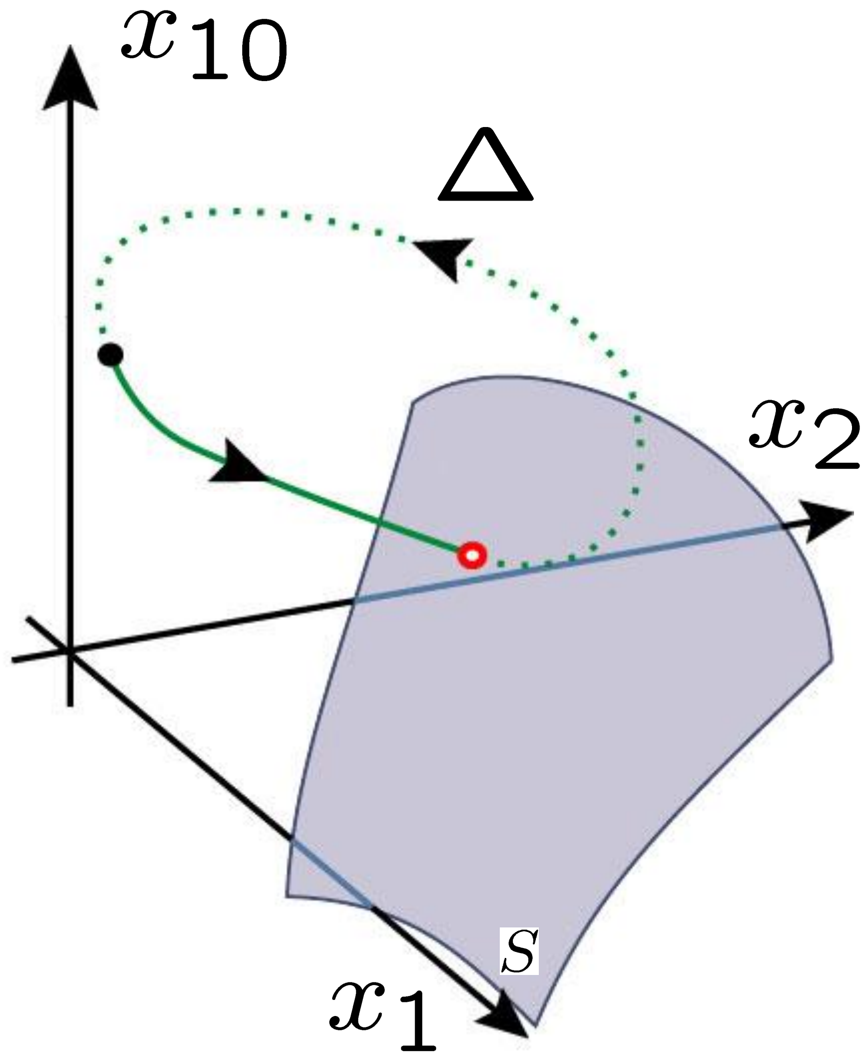
Switching Surface or Impact Surface

$$S = \{x \in \mathcal{X} \mid \varphi(x) = 0\}$$

(Hyper Surface in state space $\mathcal{X}$)

Aim: Design A Stable Periodic Orbit in a High-DOF Hybrid Model





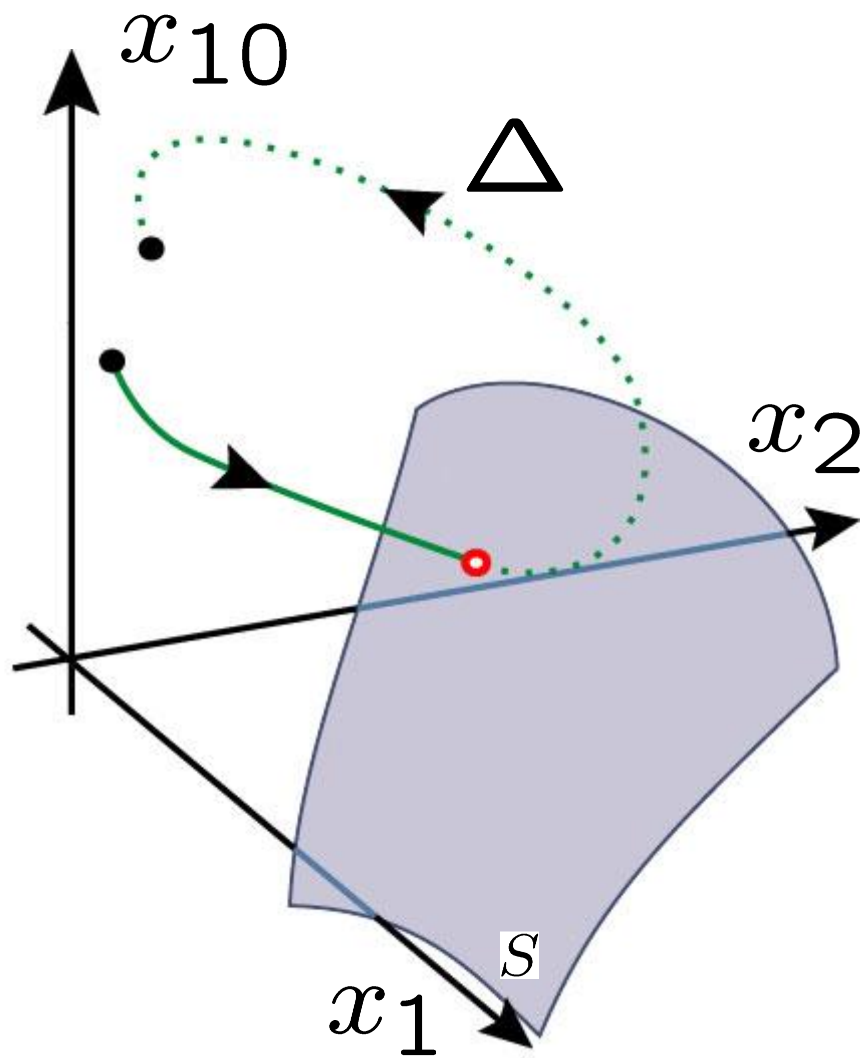
Switching Surface or Impact Surface

$$S = \{x \in \mathcal{X} \mid \varphi(x) = 0\}$$

(Hyper Surface in state space $\mathcal{X}$)

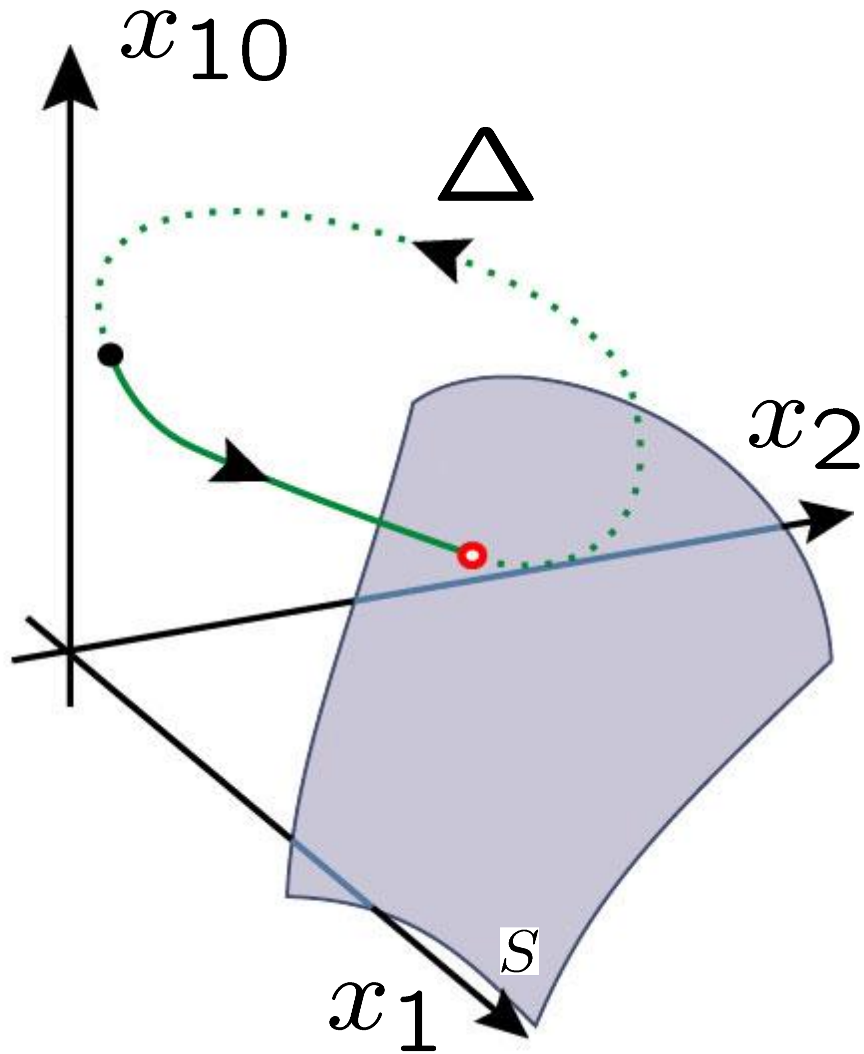
Aim: Design A Stable Periodic Orbit in a High-DOF Hybrid Model

Most solutions are not periodic!



Aim: Design A Stable Periodic Orbit in a High-DOF Hybrid Model

Harder than shown because require stability too!

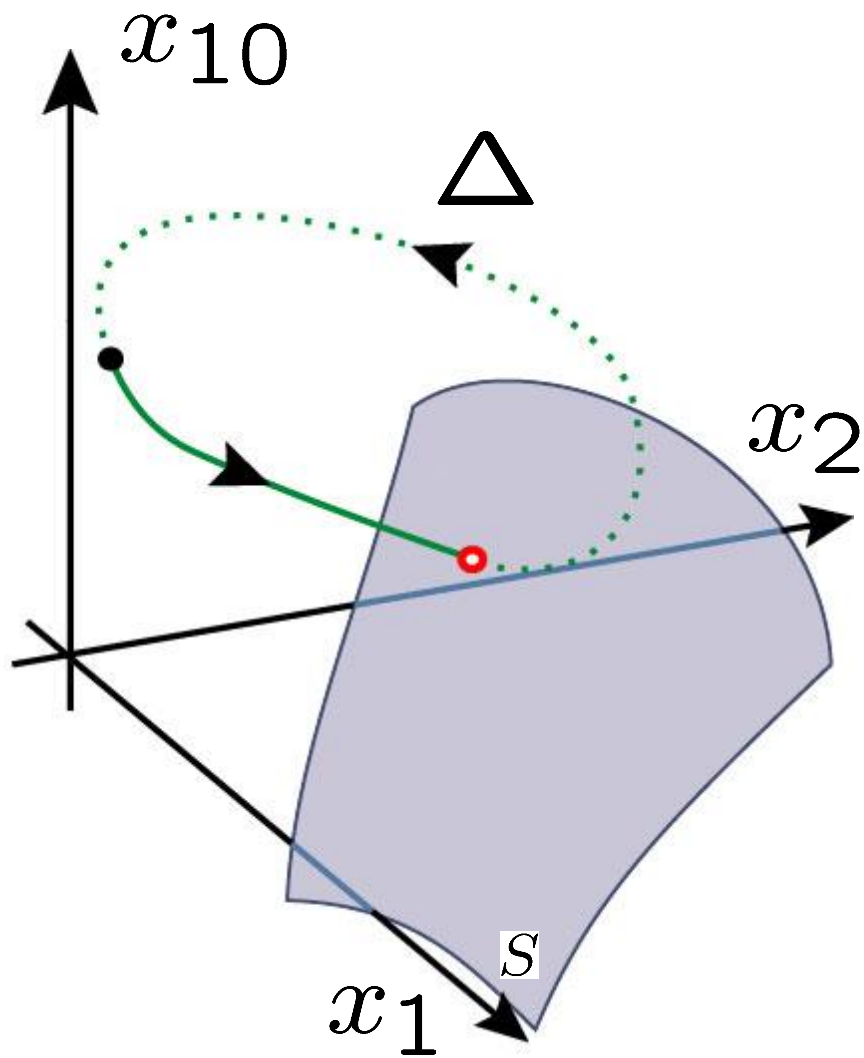


Aim: Design A Stable Periodic Orbit in a High-DOF Hybrid Model

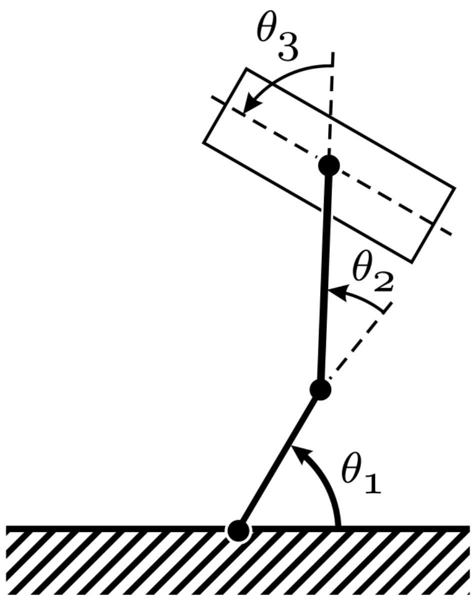
Our Approach

- **Step 1:** Use **Virtual Constraints** to reduce the complexity of the problem

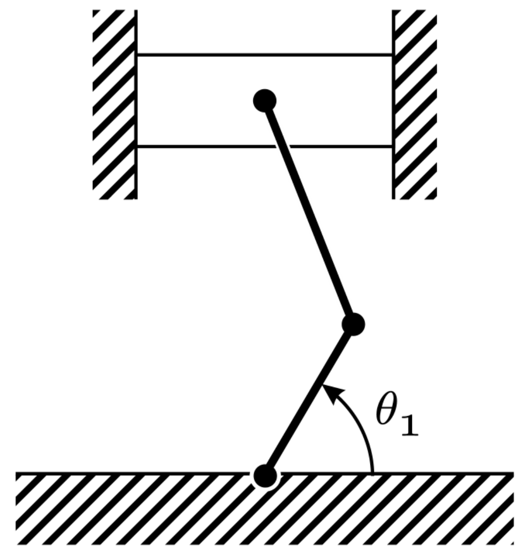
- **Step 2:** Optimize performance within the obtained feedback structure…



Idea of Virtual Constraints

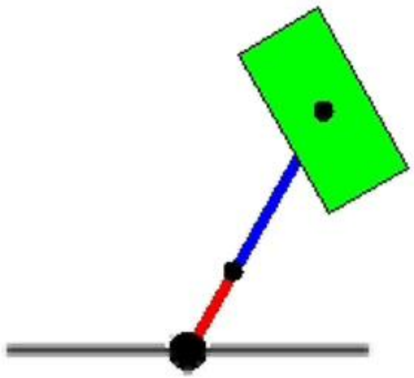


3 DOF Piston

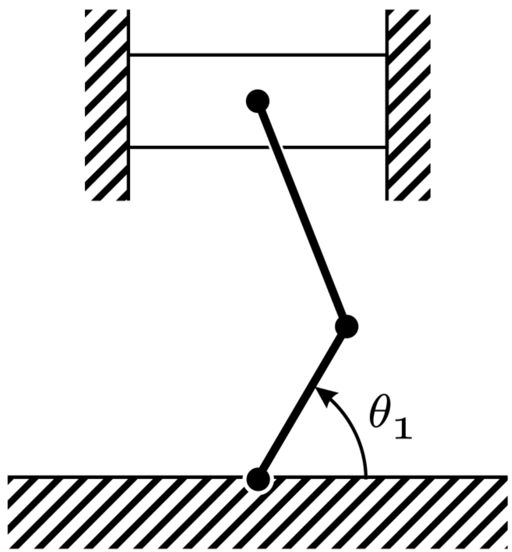


1 DOF Piston

Idea of Virtual Constraints

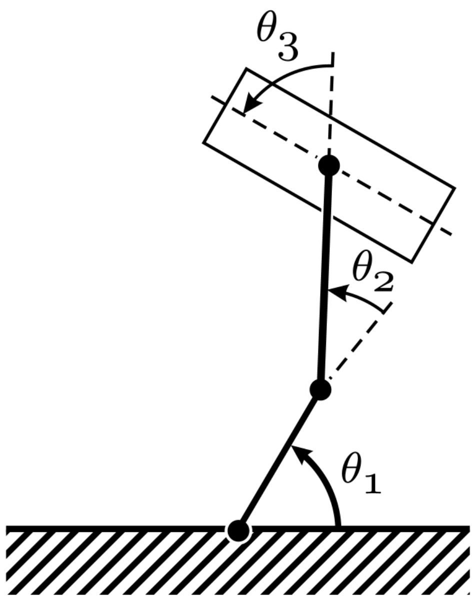


3 DOF Piston

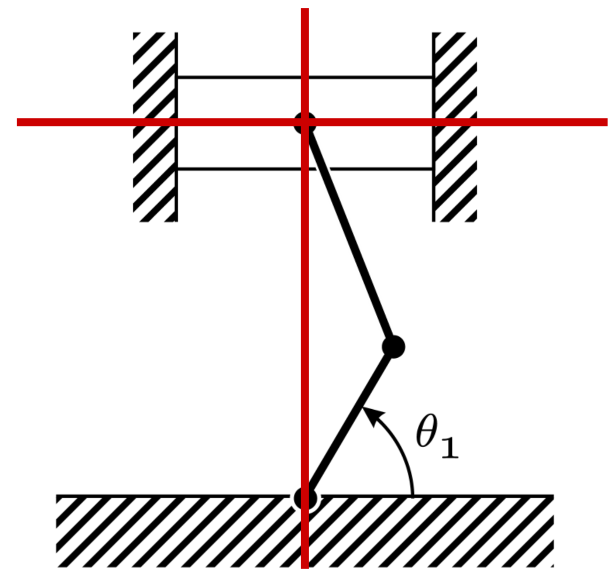


1 DOF Piston

Idea of Virtual Constraints



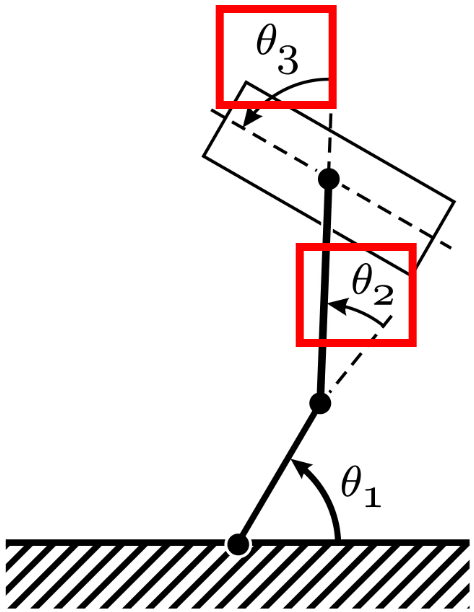
3 DOF Piston



1 DOF Piston

Cylinder walls impose 2 Constraints

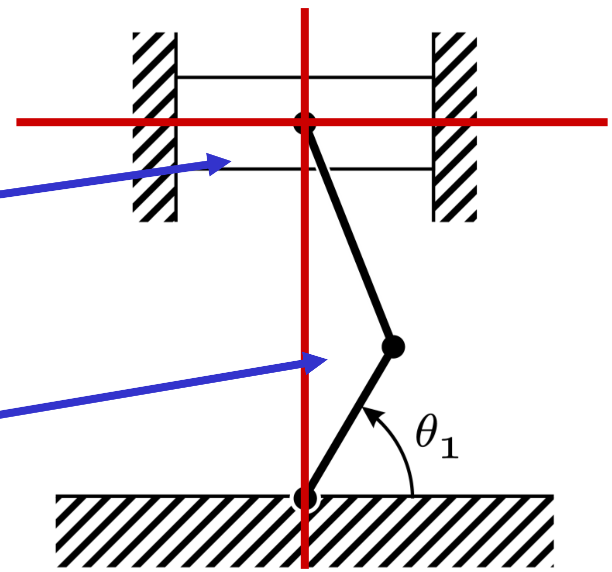
Idea of Virtual Constraints



3 DOF Piston

$\theta_3(\theta_1)$

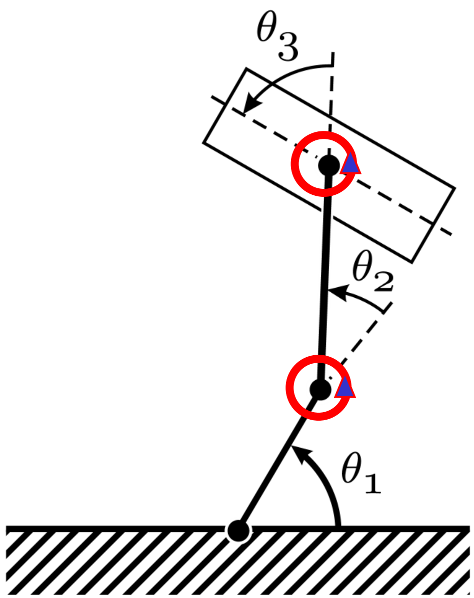
$\theta_2(\theta_1)$



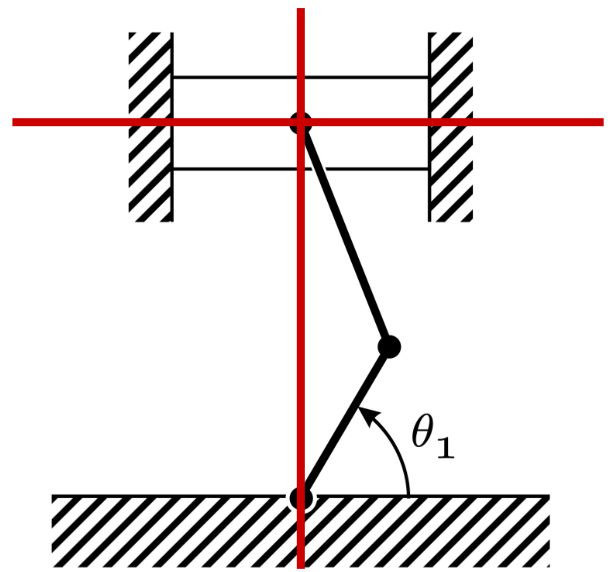
1 DOF Piston

$$\begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} \pi - \theta_1 - \arccos\left(\frac{L_1}{L_2}\cos(\theta_1)\right) \\ \arccos\left(\frac{L_1}{L_2}\cos(\theta_1)\right) \end{bmatrix}$$

Idea of Virtual Constraints



- Assume joints θ_2 and θ_3 are actuated
-



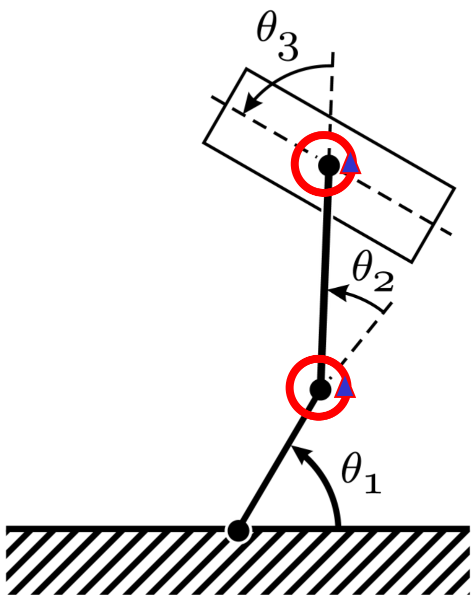
3 DOF Piston

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \theta_2 - \left(\pi - \theta_1 - \arccos\left(\frac{L_1}{L_2}\cos(\theta_1)\right)\right) \\ \theta_3 - \arccos\left(\frac{L_1}{L_2}\cos(\theta_1)\right) \end{bmatrix}$$

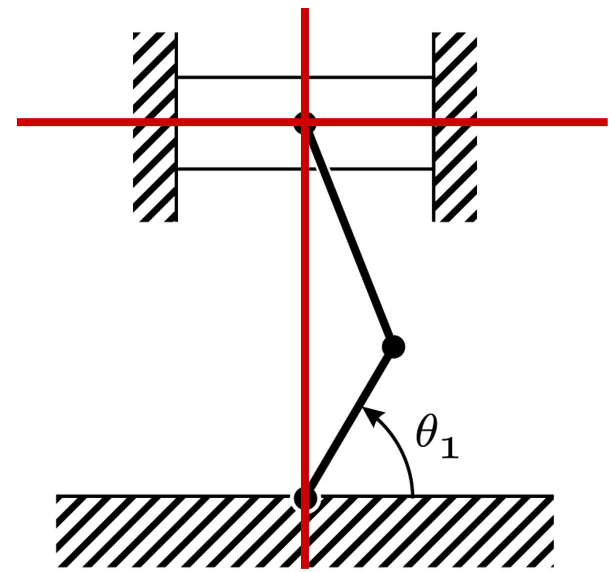
1 DOF Piston

$$\begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} \pi - \theta_1 - \arccos\left(\frac{L_1}{L_2}\cos(\theta_1)\right) \\ \arccos\left(\frac{L_1}{L_2}\cos(\theta_1)\right) \end{bmatrix}$$

Idea of Virtual Constraints



- Assume joints θ_2 and θ_3 are actuated
- Use feedback to impose constraints



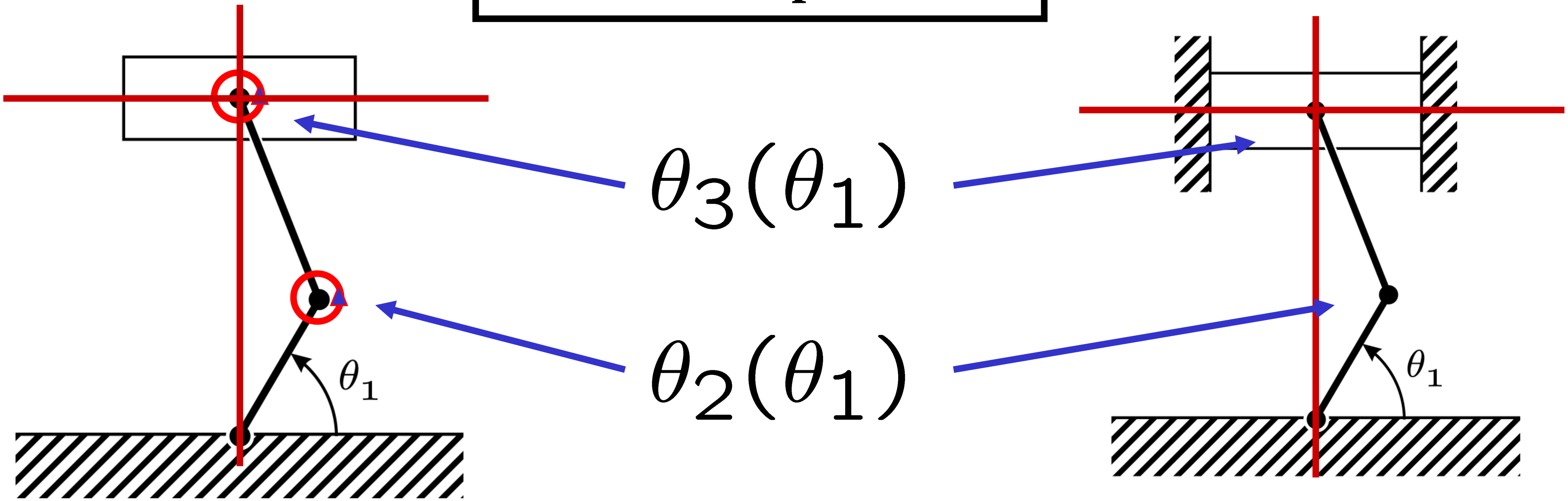
3 DOF Piston

1 DOF Piston

$$\lim_{t\to\infty} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \longleftrightarrow \quad \begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} \pi - \theta_1 - \arccos\left(\frac{L_1}{L_2}\cos(\theta_1)\right) \\ \arccos\left(\frac{L_1}{L_2}\cos(\theta_1)\right) \end{bmatrix}$$

Idea of Virtual Constraints

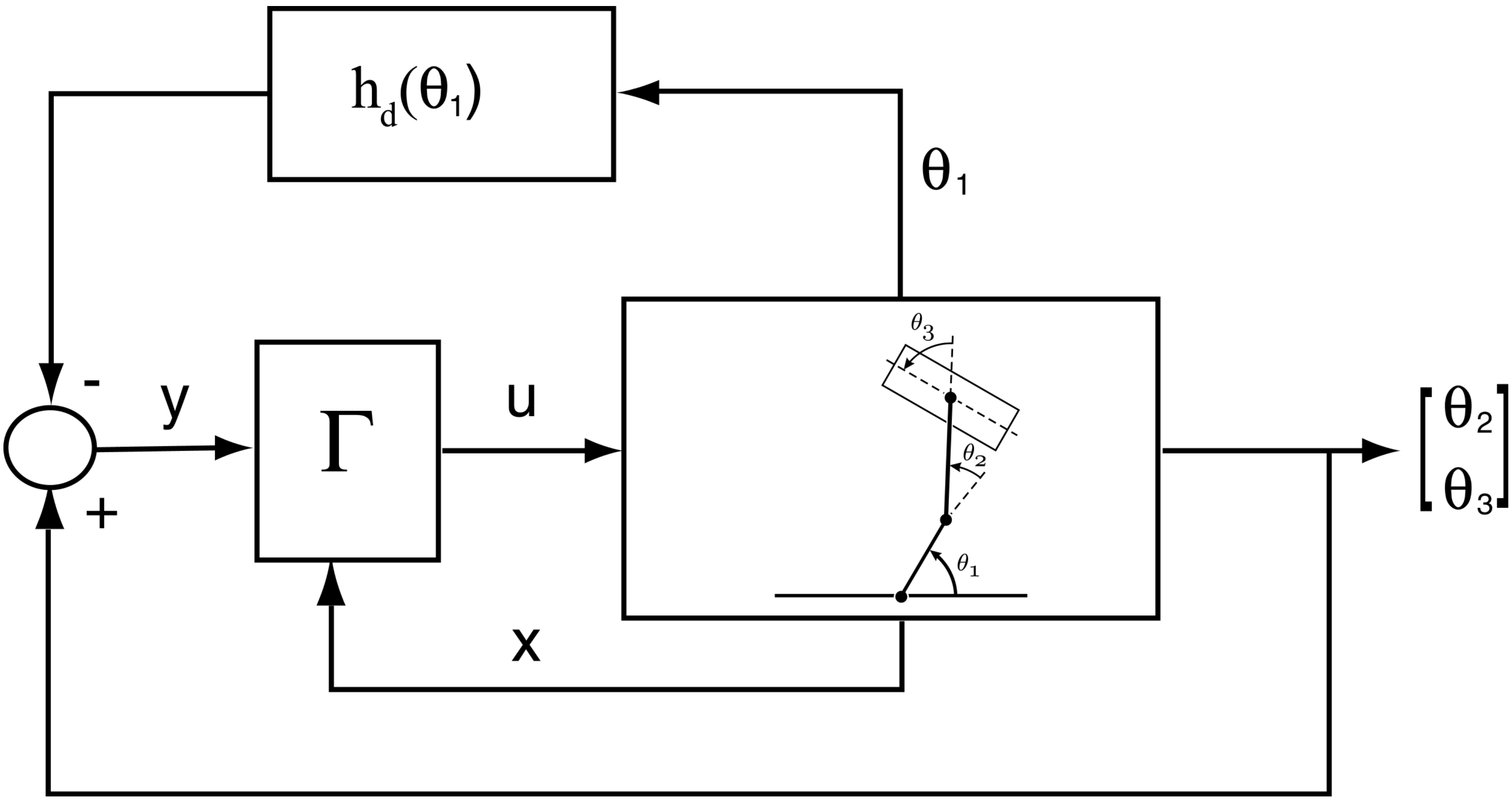
Asymptotic behavior is a 1 DOF piston



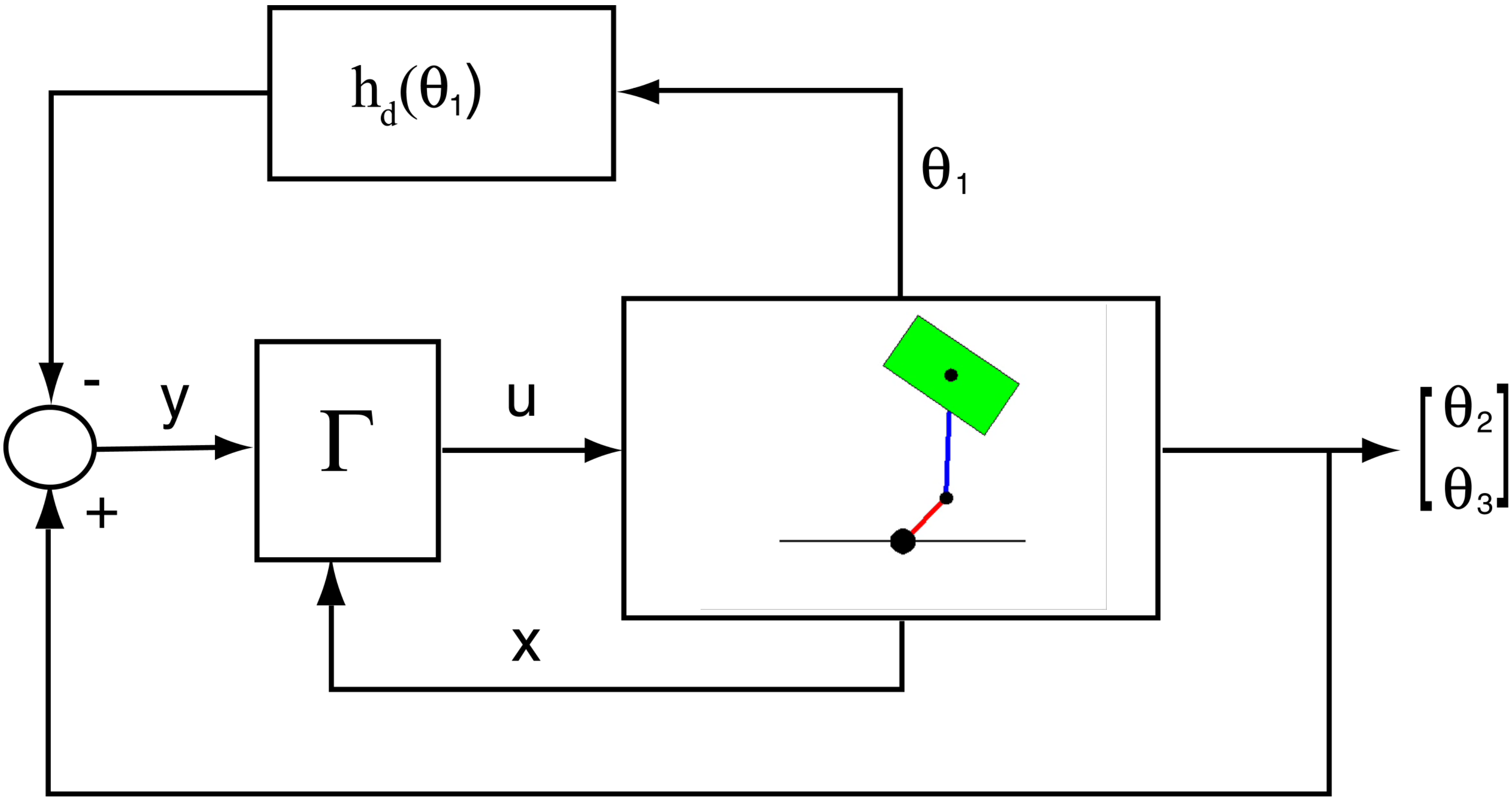
3 DOF Piston with 2 Actuators **1 DOF Piston**

$$\lim_{t\to\infty}\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \longleftrightarrow \quad \begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} \pi - \theta_1 - \arccos\left(\frac{L_1}{L_2}\cos(\theta_1)\right) \\ \arccos\left(\frac{L_1}{L_2}\cos(\theta_1)\right) \end{bmatrix}$$

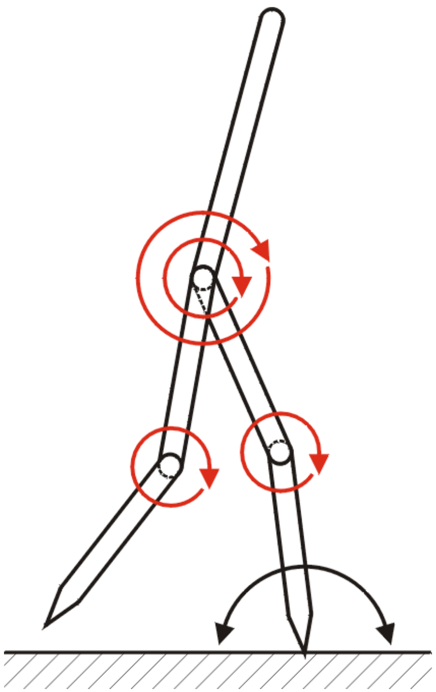
Idea of Virtual Constraints



Idea of Virtual Constraints



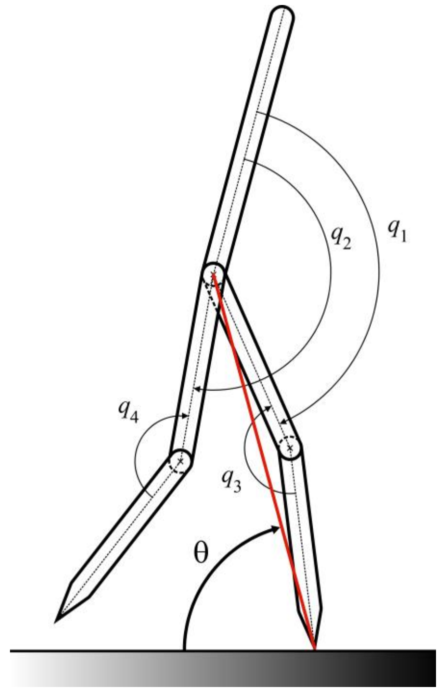
Idea of Virtual Constraints



5 DOF Robot

Four Virtual Constraints

$$\begin{bmatrix} y_1 \\ \vdots \\ y_4 \end{bmatrix} = \begin{bmatrix} q_1 - h_{d,1}(\theta) \\ \vdots \\ q_4 - h_{d,4}(\theta) \end{bmatrix}$$

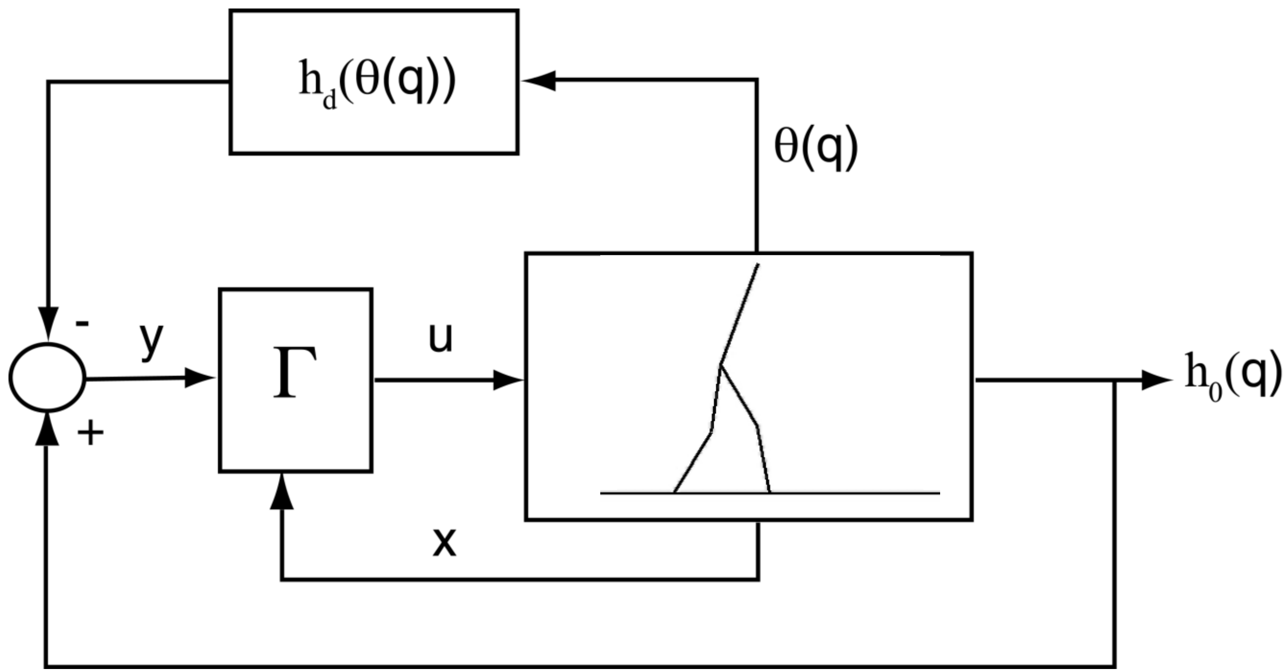


1 Un-Actuated DOF

Asymptotic behavior is a 1 DOF robot

Idea of Virtual Constraints

$$y = h(q) = h_0(q) - h_d(\theta(q))$$

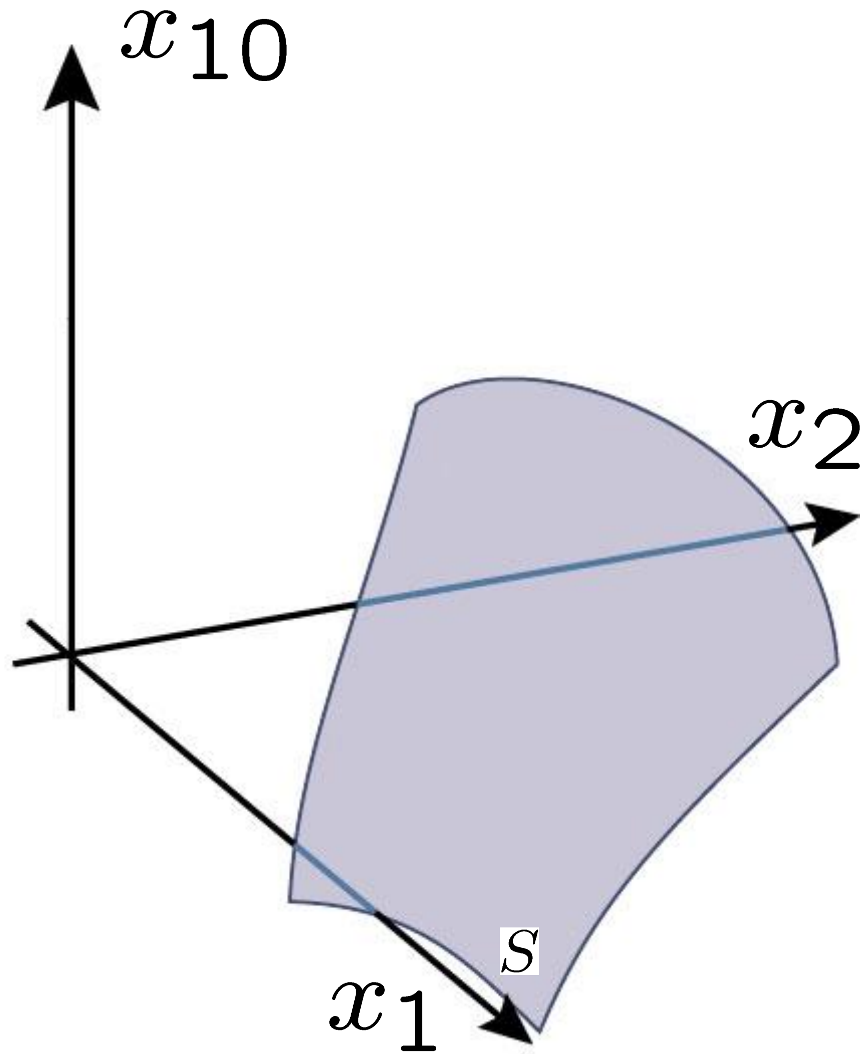


For “posture principles”, see [Kajita et al., ’92; Hurmuzlu, ’93; Ohno ‘01]

Complexity Reduction Through (Hybrid)-Invariance and Attractivity

Virtual Constraints in ODE model

$$\dot{x} \;=\; f(x) + g(x)u$$

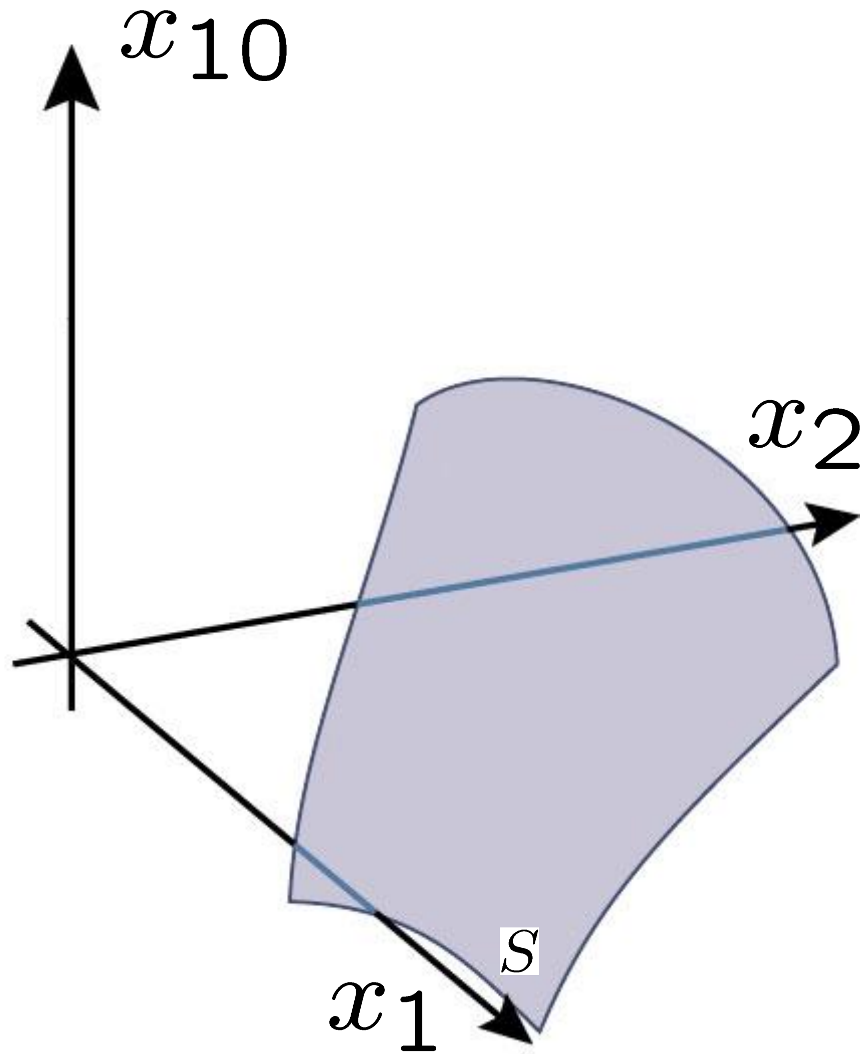


Complexity Reduction Through (Hybrid)-Invariance and Attractivity

Virtual Constraints in ODE model

$$\dot{x} = f(x) + g(x)u$$

$$y = h(q) = h_0(q) - h_d(\theta(q)) \in R^4$$

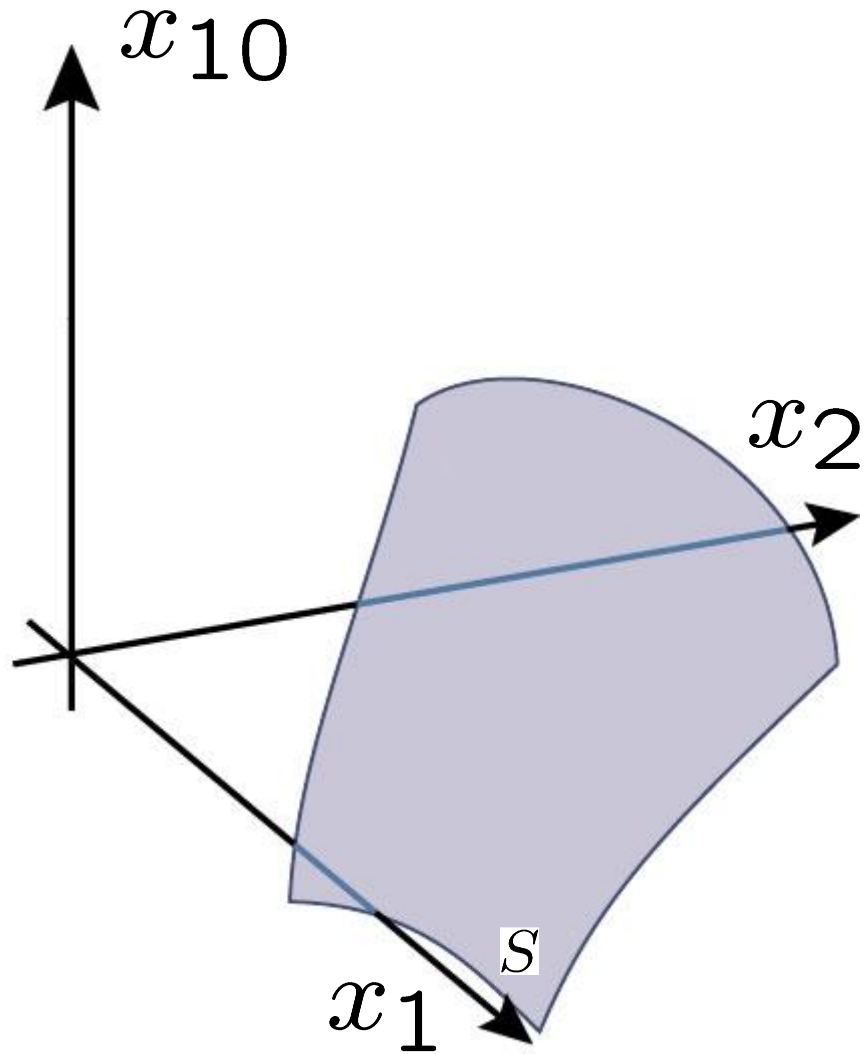


Complexity Reduction Through (Hybrid)-Invariance and Attractivity

Virtual Constraints in ODE model

$$\dot{x} = f(x) + g(x)u^*(x)$$
$$y = h(q) = h_0(q) - h_d(\theta(q)) \in R^4$$

Design: $u^*(x) \quad s.t. \quad y(t) \to 0$

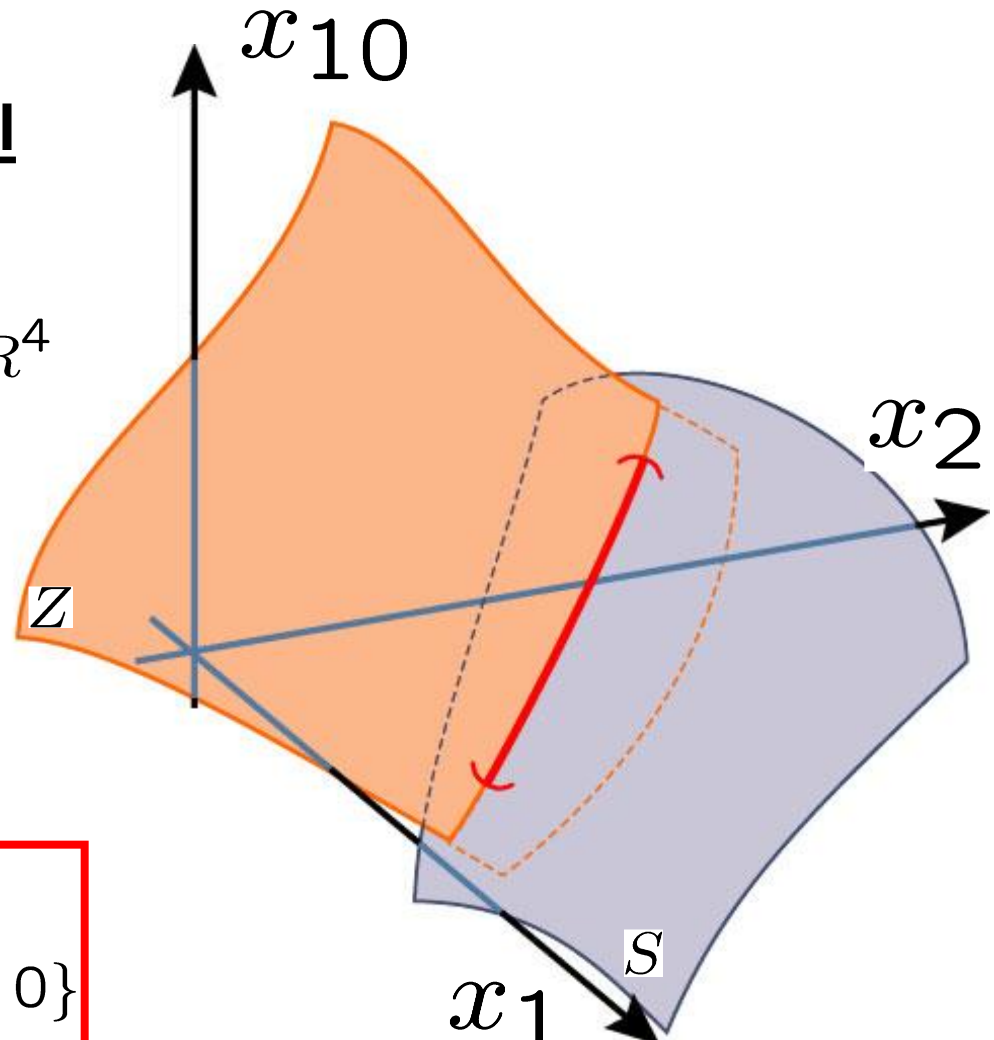


Complexity Reduction Through (Hybrid)-Invariance and Attractivity

Virtual Constraints in ODE model

$$\dot{x} = f(x) + g(x)u^*(x)$$
$$y = h(q) = h_0(q) - h_d(\theta(q)) \in R^4$$

Design: $u^*(x) \quad s.t. \quad y(t) \to 0$



Create: 2-dim. invariant surface:
$Z = \{(q, \dot{q}) \mid y(q) = 0 \ \& \ \dot{y}(q, \dot{q}) = 0\}$

Complexity Reduction Through (Hybrid)-Invariance and Attractivity

Virtual Constraints in ODE model

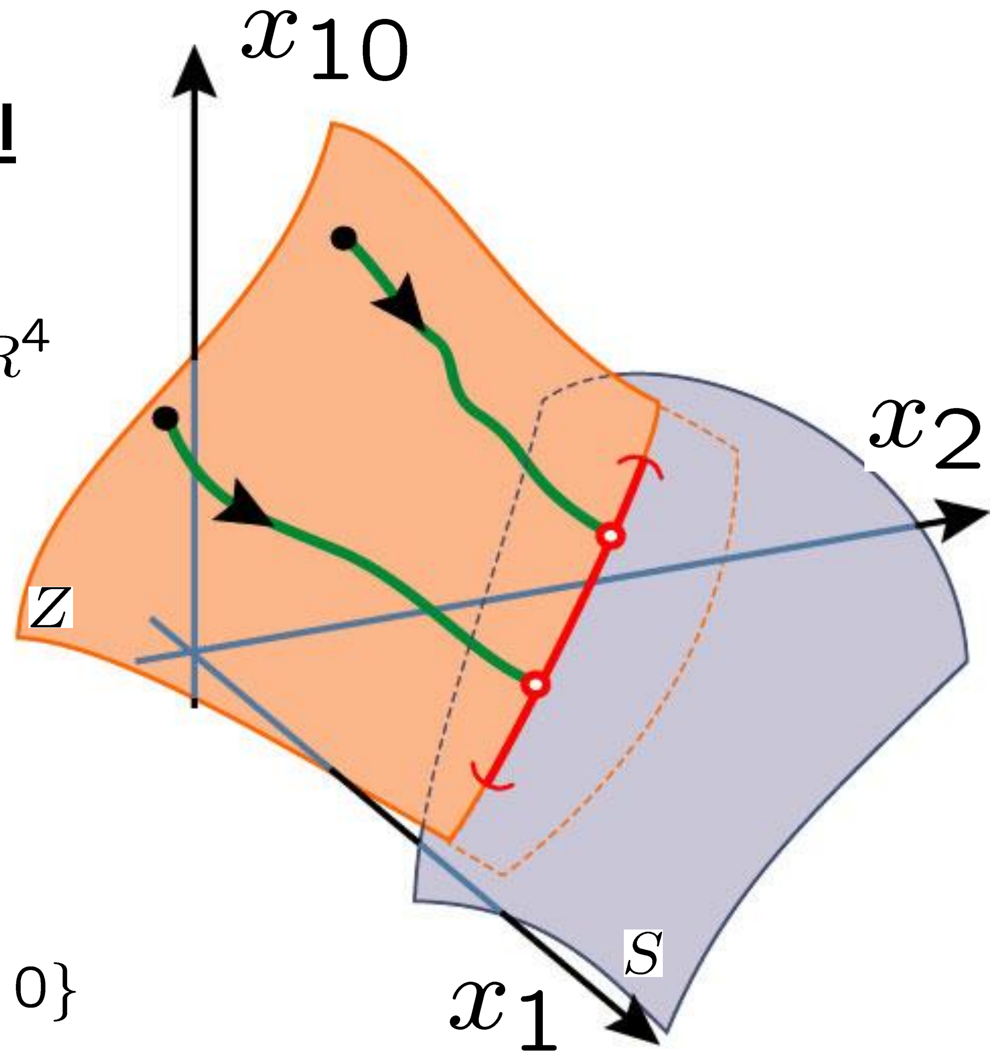
$$\dot{x} = f(x) + g(x)u^*(x)$$
$$y = h(q) = h_0(q) - h_d(\theta(q)) \in R^4$$

Design: $u^*(x) \quad s.t. \quad y(t) \to 0$

Byrnes-Isidori Zero Dynamics

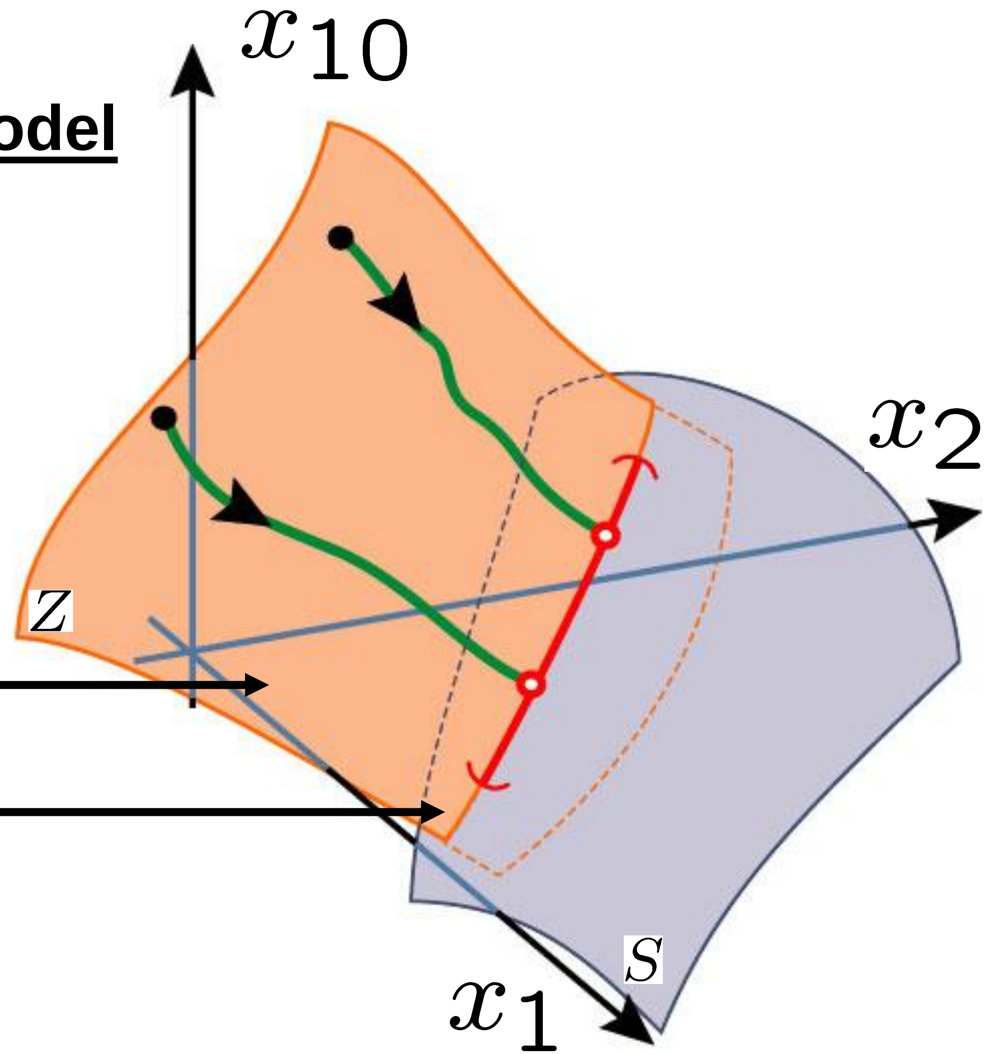
Create: 2-dim. invariant surface:

$Z = \{(q, \dot{q}) \mid y(q) = 0 \ \& \ \dot{y}(q, \dot{q}) = 0\}$



Complexity Reduction Through (Hybrid)-Invariance and Attractivity

Virtual Constraints in IMPACT model



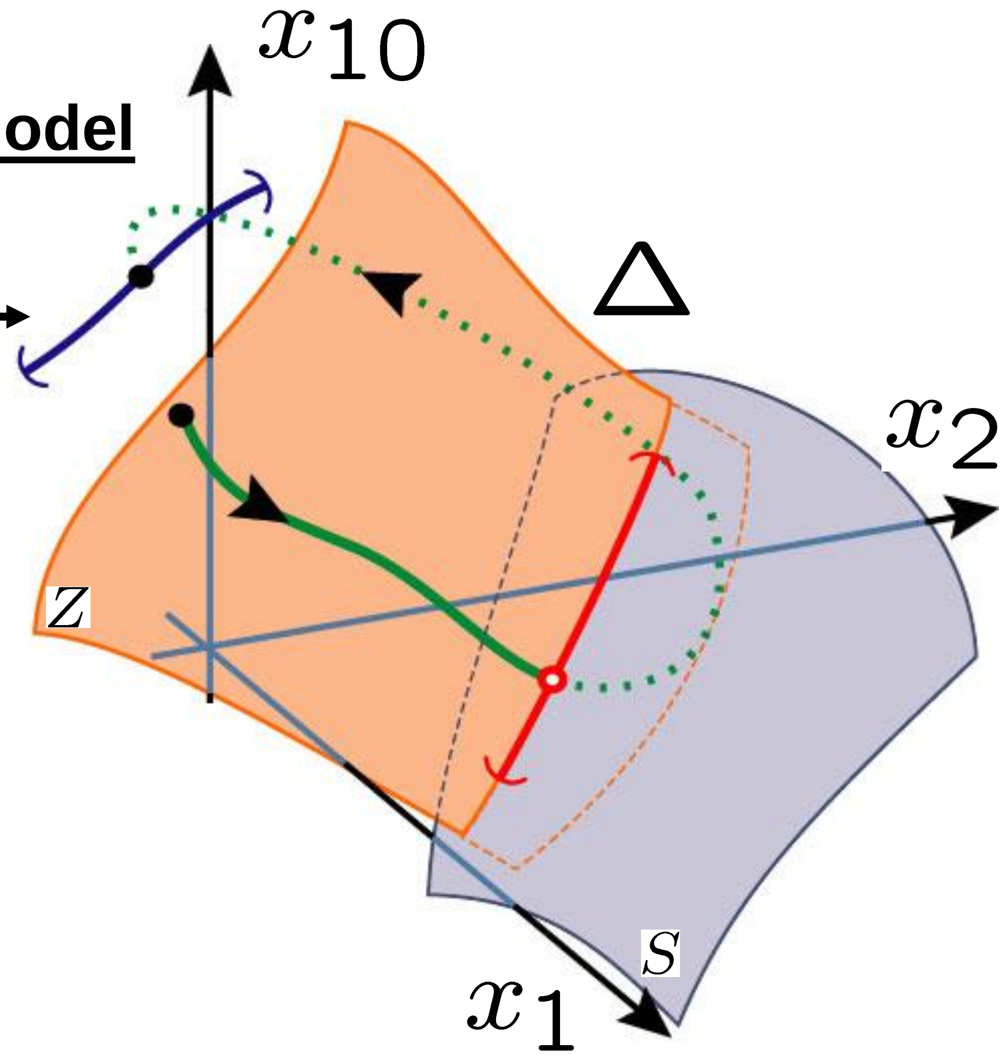
$Z =$ 2-dim. (invariant) surface

$S \cap Z =$ 1-dim. (smooth) surface

Complexity Reduction Through (Hybrid)-Invariance and Attractivity

Virtual Constraints in IMPACT model

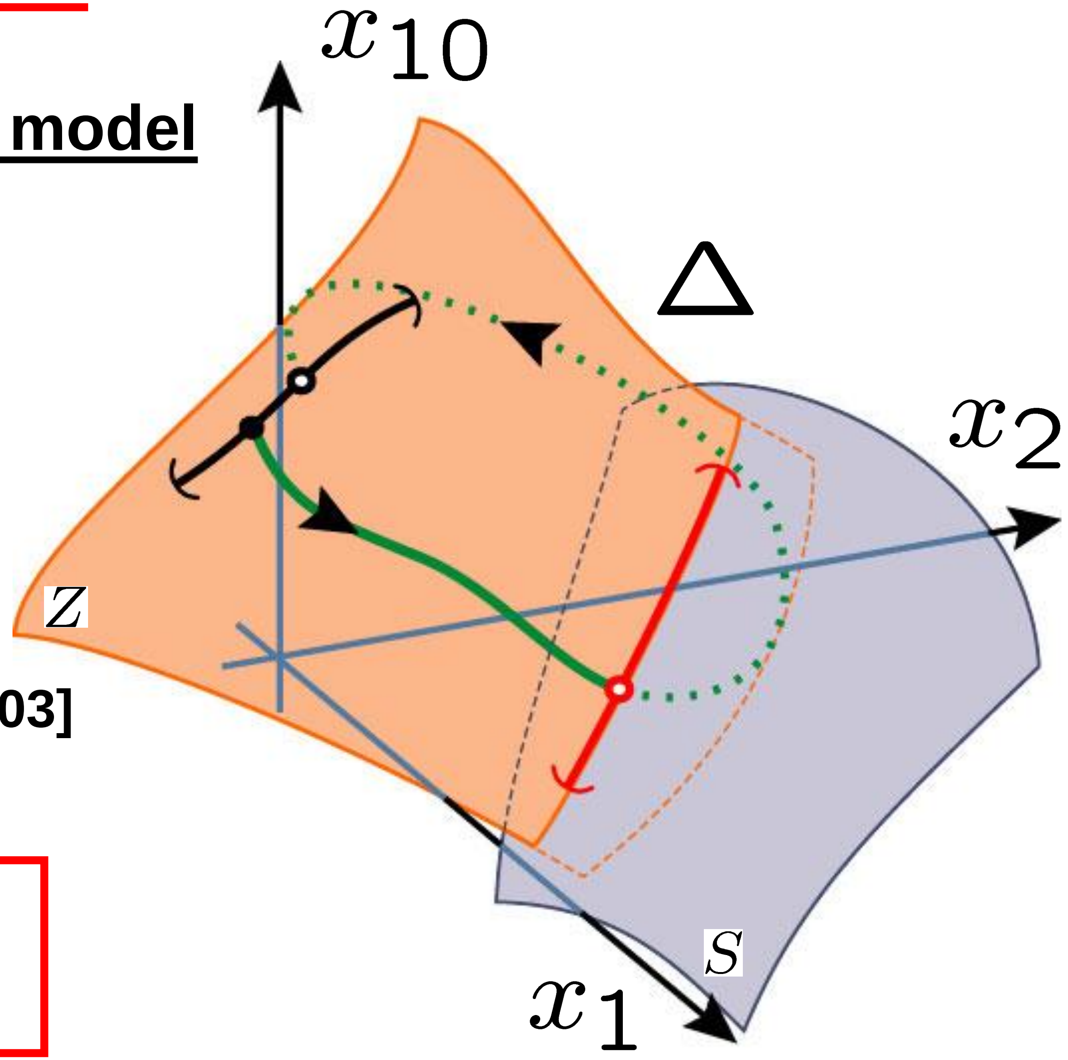
In general $\Delta : S \cap Z \not\subset Z$



Complexity Reduction Through (Hybrid)-Invariance and Attractivity

Virtual Constraints in IMPACT model

In general $\Delta : S \cap Z \not\subset Z$



Theorem **[Westervelt's Thesis-2003]**

Can design surface such that

$$\Delta : S \cap Z \subset Z$$

Complexity Reduction Through (Hybrid)-Invariance and Attractivity

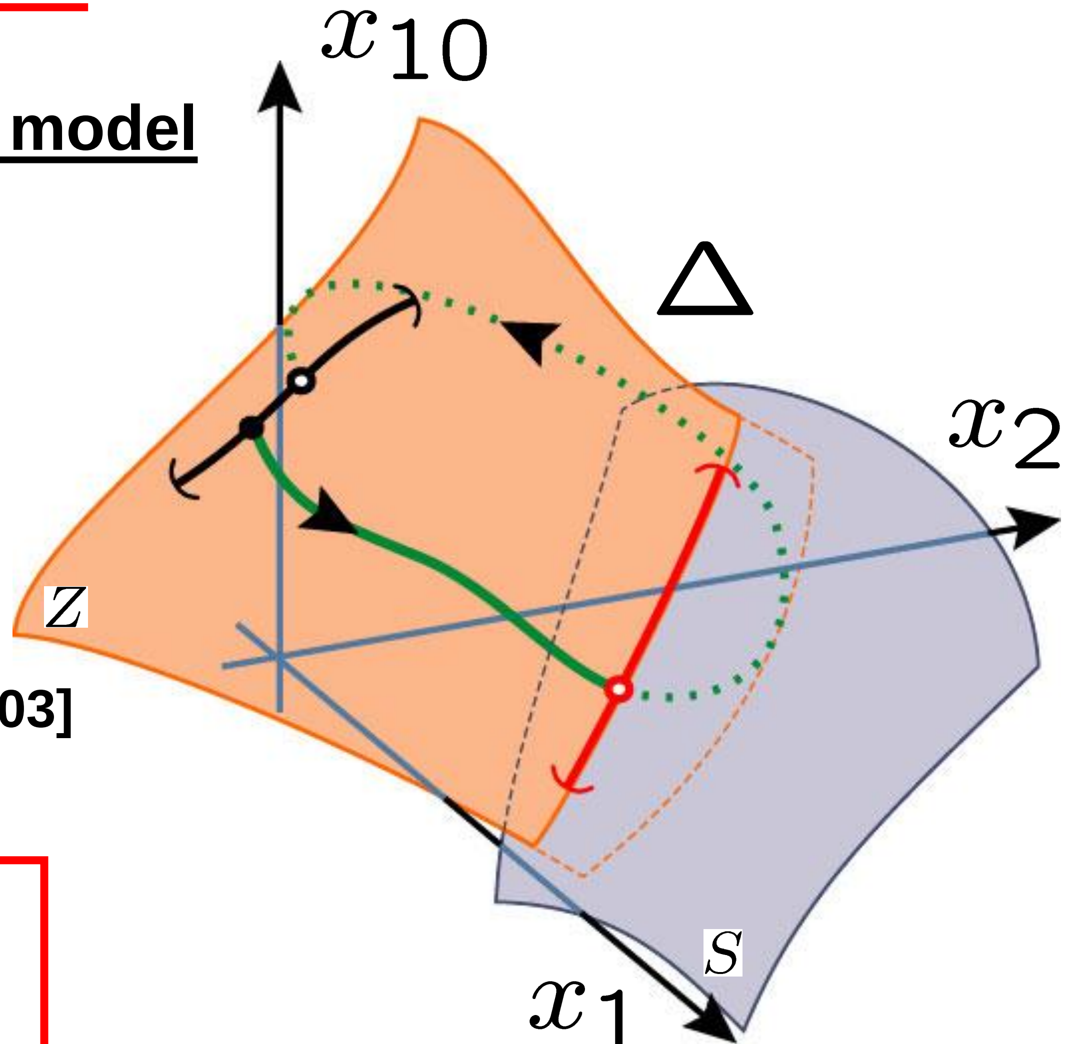
Virtual Constraints in IMPACT model

HYBRID ZERO DYNAMICS

Theorem [Westervelt's Thesis-2003]

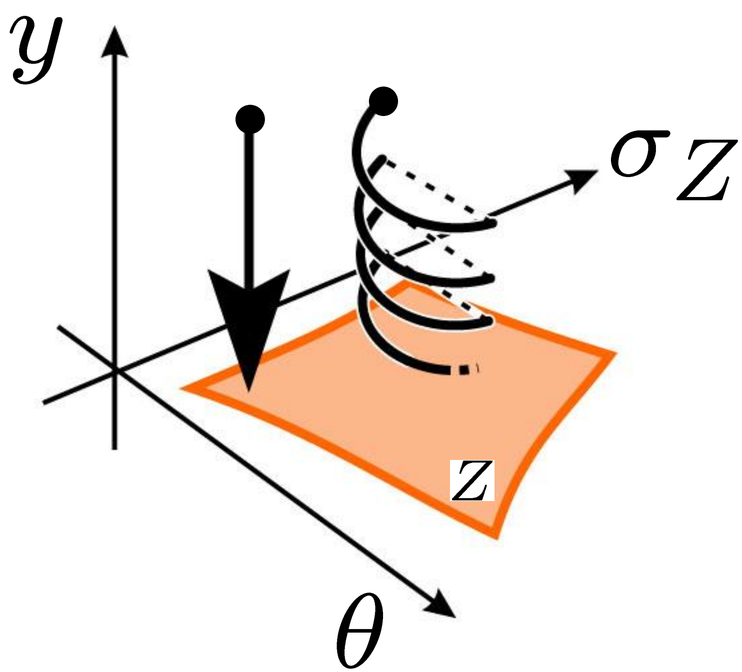
Can design surface such that

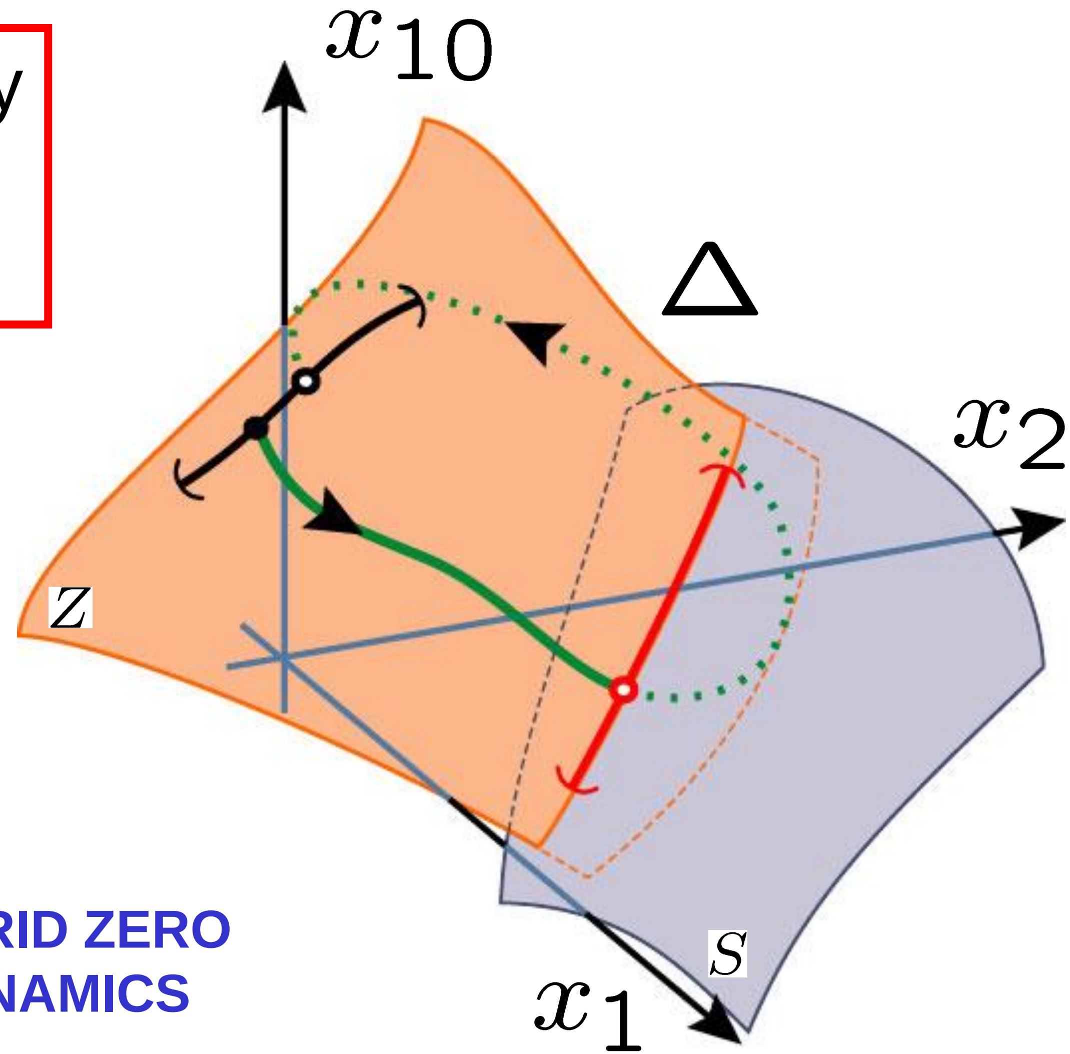
$$\Delta : S \cap Z \subset Z$$



Complexity Reduction Through (Hybrid)-Invariance and Attractivity

Render surface sufficiently attractive to overcome impulse "disturbance" Δ



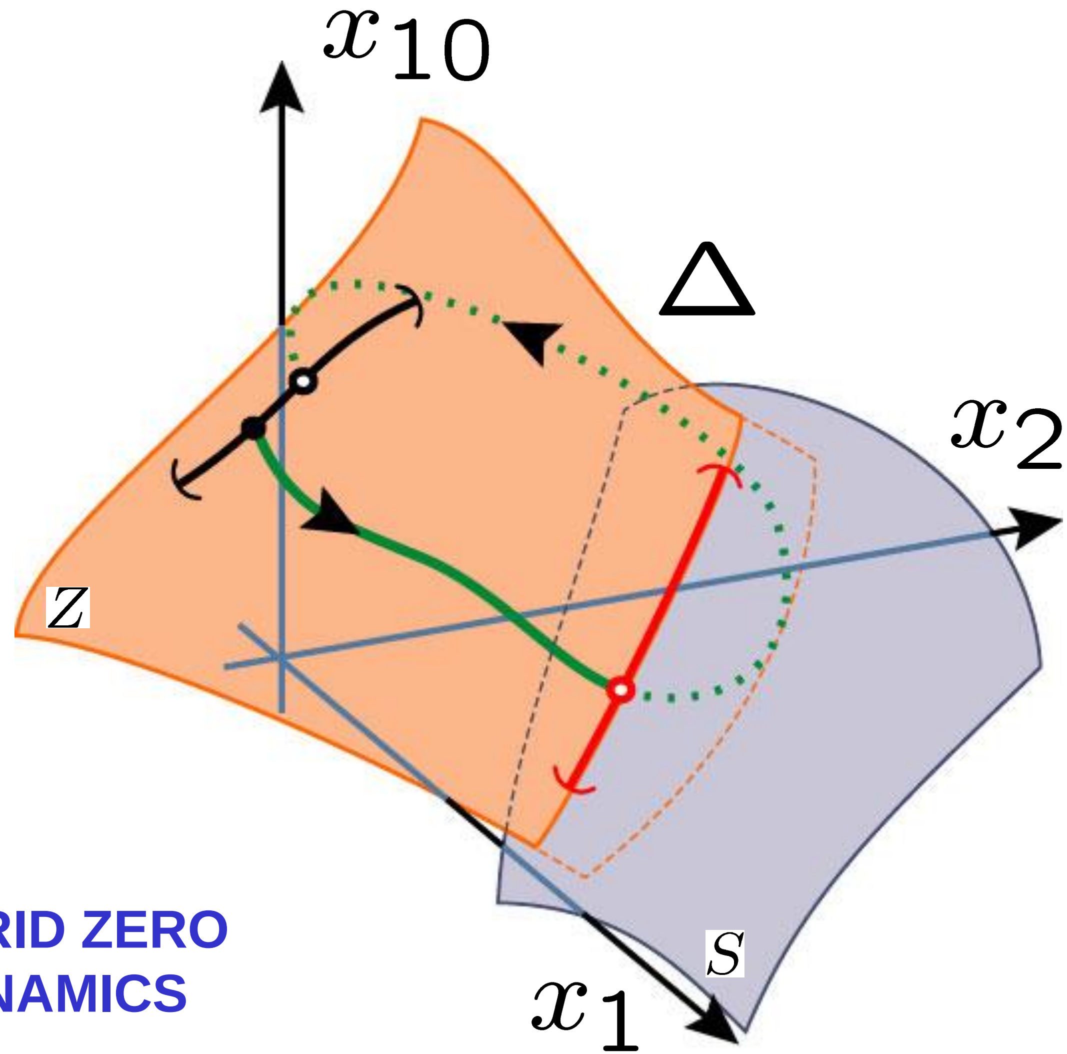


HYBRID ZERO DYNAMICS

Hybrid Zero Dynamics for Bipeds

Can do Design on the Basis of 1 DOF Hybrid Sub-Dynamic!

$$\dot{z} = f_{zero}(z), \quad z^- \notin S \cap Z$$
$$z^+ = \Delta(z^-), \quad z^- \in S \cap Z$$



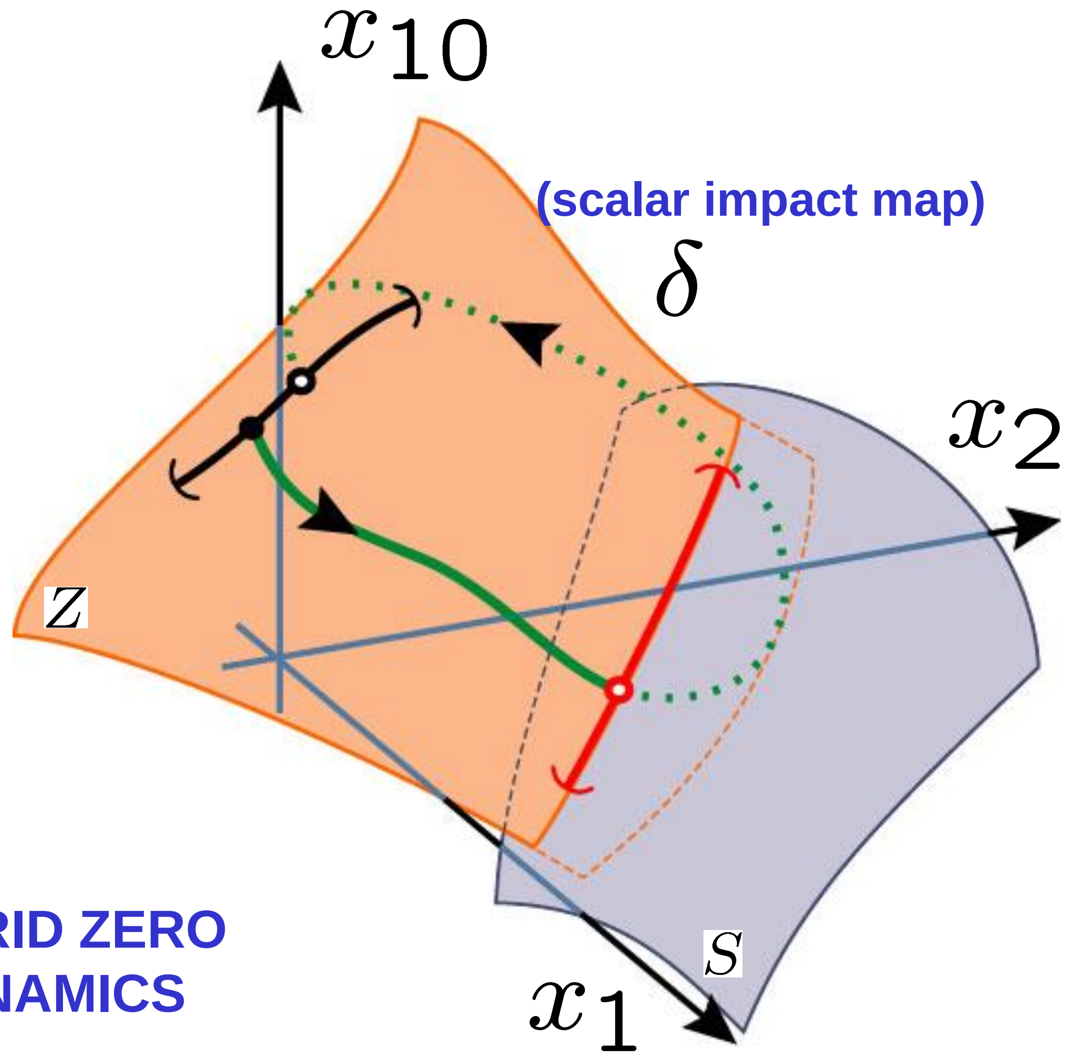
HYBRID ZERO DYNAMICS

Hybrid Zero Dynamics for Bipeds

Can do Design on the Basis of 1 DOF Hybrid Sub-Dynamic!

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} \kappa_1(\xi_1)\xi_2 \\ \kappa_2(\xi_1) \end{bmatrix}$$

$$\xi_2^+ = \delta_{zero}\xi_2^-$$



HYBRID ZERO DYNAMICS

Hybrid Zero Dynamics Analysis

$$L_{zero} = K_{zero} - V_{zero} = \text{Lagrangian of swing phase model}$$

$$\frac{d}{dt}\frac{\partial L_{zero}}{\partial \dot{\theta}} - \frac{\partial L_{zero}}{\partial \theta} = 0 \quad \Rightarrow \quad \begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} \kappa_1(\xi_1)\xi_2 \\ \kappa_2(\xi_1) \end{bmatrix}$$

$$L_{zero} = \underbrace{\frac{1}{2}\left(\frac{\dot{\xi}_1}{\kappa_1(\xi_1)}\right)^2}_{\text{Kinetic Energy}} - \underbrace{\left(\int_{\theta^+}^{\xi_1} -\frac{\kappa_2(\xi)}{\kappa_1(\xi)}\, d\xi\right)}_{\text{Potential Energy}}$$

Hybrid Zero Dynamics Analysis

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} \kappa_1(\xi_1)\xi_2 \\ \kappa_2(\xi_1) \end{bmatrix}$$

$$\xi_2^+ = \delta_{zero} \xi_2^-$$ (impact map on Z)

Theorem: [**Westervelt's Thesis-2003**] There exists an exponentially stable periodic orbit of the hybrid zero dynamics if, and only if,

a) $(\delta_{zero})^2 < 1$ (energy loss at impact)

b) $$\frac{\delta_{zero}^2}{1-\delta_{zero}^2} V_{zero}(\theta^-) + V_{\max} < 0$$ (evolution of energy during SS)

Theorem: [**Grizzle-Abba-Plestan 2001**] Above orbit is asymptotically stabilizable in the full-order model.

How to Use for Controller Design

- Finitely parametrize the outputs: $y = h_a(q) = h_0(q) - h_d(q, \boxed{a})$
- Impose invariance condition:

$$h_a \circ \Delta\big|_{(S \cap Z_a)} = 0$$
$$L_f h_a \circ \Delta\big|_{(S \cap Z_a)} = 0$$

- Stability guaranteed if, and only if, two inequalities hold

a) $(\delta_{zero})^2 < 1$

b) $\dfrac{\delta_{zero}^2}{1-\delta_{zero}^2} V_{zero}(\theta^-) + V_{\max} < 0$

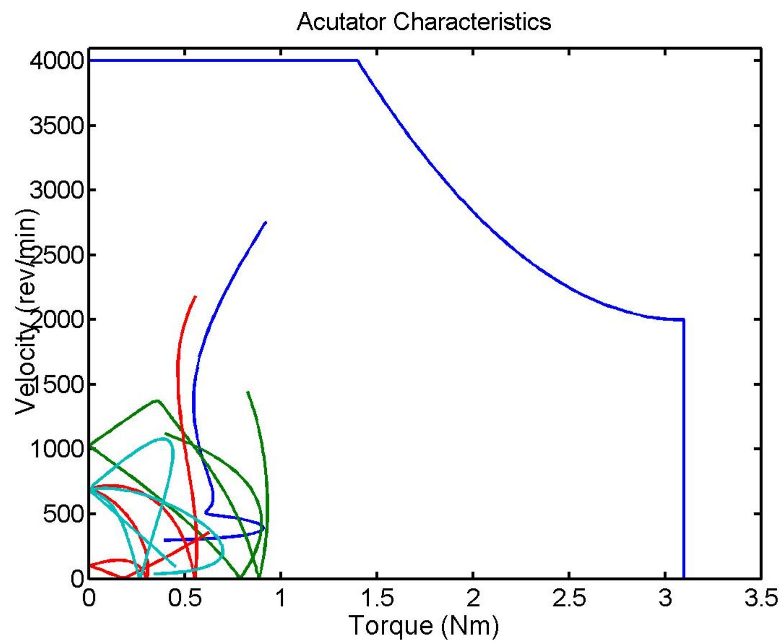
How to Use for Controller Design

- Achieve performance by tuning parameters via optimization on 2-dimensional model, subject to previous constraints.

$$\dot{z} = f_{a,zero}(z), \quad z^- \notin S \cap Z_a$$
$$z^+ = \Delta(z^-), \quad z^- \in S \cap Z_a$$

$$J(a) := \frac{1}{p_2^h(T^-)} \int_0^{T^-} \left\| u_a^*(t) \right\|^2 dt$$

- **Can also include contact constraints**
- **They can be written as affine functions of the (squared) velocity**
- **Actuator limitations, etc.**

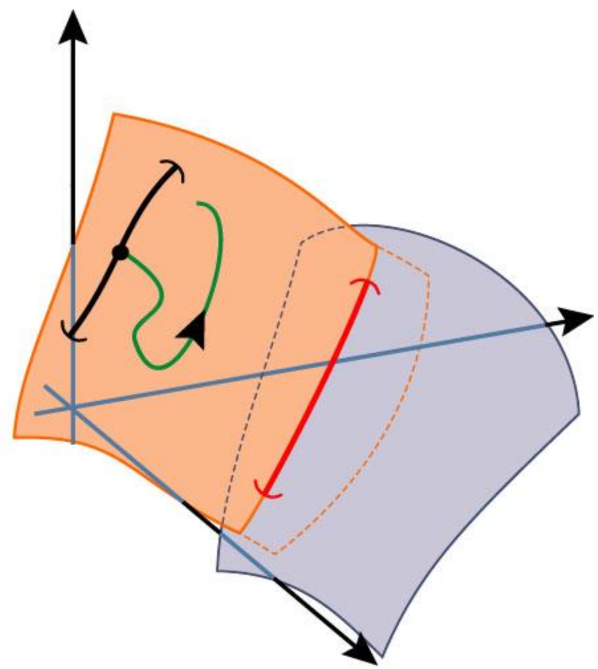


How to Use for Controller Design

- Achieve performance by tuning parameters via optimization on 2-dimensional model, subject to previous constraints.

$$\dot{z} = f_{a,zero}(z), \quad z^- \notin S \cap Z_a$$
$$z^+ = \Delta(z^-), \quad z^- \in S \cap Z_a$$

$$J(a) := \frac{1}{p_2^h(T^-)} \int_0^{T^-} \left\| u_a^*(t) \right\|^2 dt$$

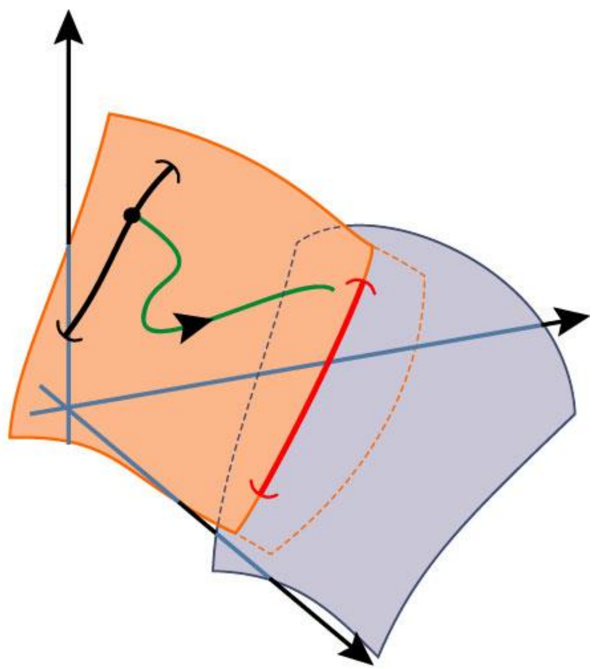


How to Use for Controller Design

- Achieve performance by tuning parameters via optimization on 2-dimensional model, subject to previous constraints.

$$\dot{z} = f_{a,zero}(z), \quad z^- \notin S \cap Z_a$$
$$z^+ = \Delta(z^-), \quad z^- \in S \cap Z_a$$

$$J(a) := \frac{1}{p_2^h(T^-)} \int_0^{T^-} \left\| u_a^*(t) \right\|^2 dt$$

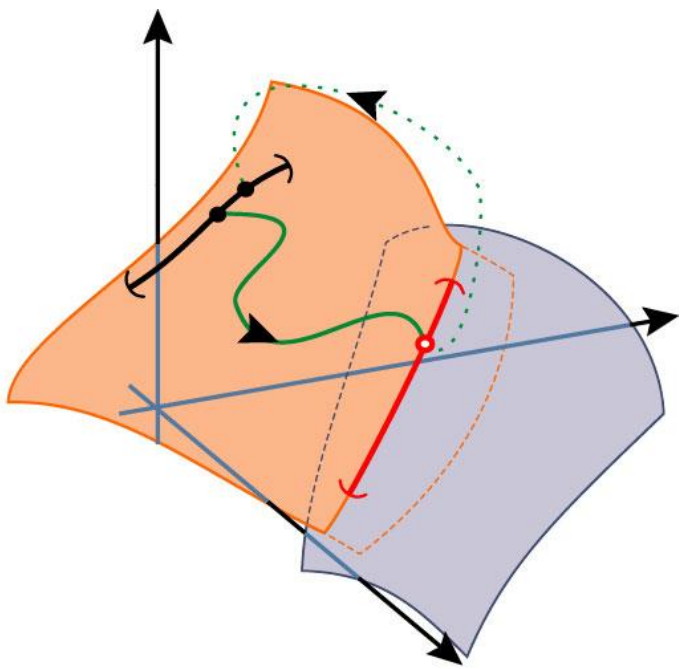


How to Use for Controller Design

- Achieve performance by tuning parameters via optimization on 2-dimensional model, subject to previous constraints.

$$\dot{z} = f_{a,zero}(z), \quad z^- \notin S \cap Z_a$$
$$z^+ = \Delta(z^-), \quad z^- \in S \cap Z_a$$

$$J(a) := \frac{1}{p_2^h(T^-)} \int_0^{T^-} \left\| u_a^*(t) \right\|^2 dt$$

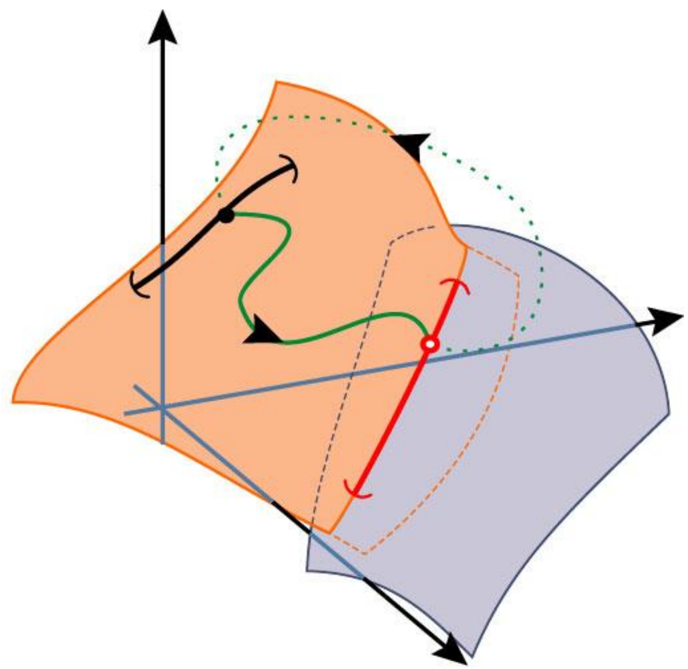


How to Use for Controller Design

- Achieve performance by tuning parameters via optimization on 2-dimensional model, subject to previous constraints.

$$\dot{z} = f_{a,zero}(z), \quad z^- \notin S \cap Z_a$$
$$z^+ = \Delta(z^-), \quad z^- \in S \cap Z_a$$

$$J(a) := \frac{1}{p_2^h(T^-)} \int_0^{T^-} \left\| u_a^*(t) \right\|^2 dt$$

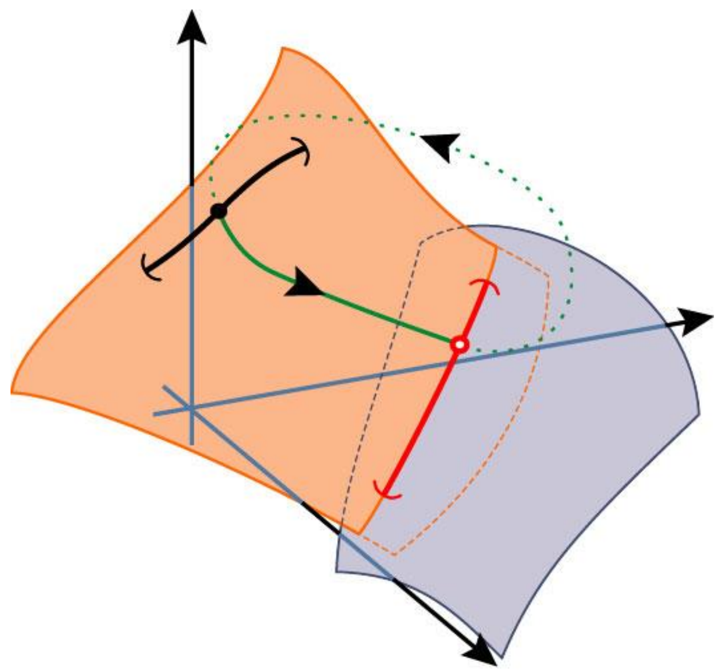


How to Use for Controller Design

- Achieve performance by tuning parameters via optimization on 2-dimensional model, subject to previous constraints.

$$\dot{z} = f_{a,zero}(z), \quad z^- \notin S \cap Z_a$$
$$z^+ = \Delta(z^-), \quad z^- \in S \cap Z_a$$

$$J(a) := \frac{1}{p_2^h(T^-)} \int_0^{T^-} \left\| u_a^*(t) \right\|^2 dt$$

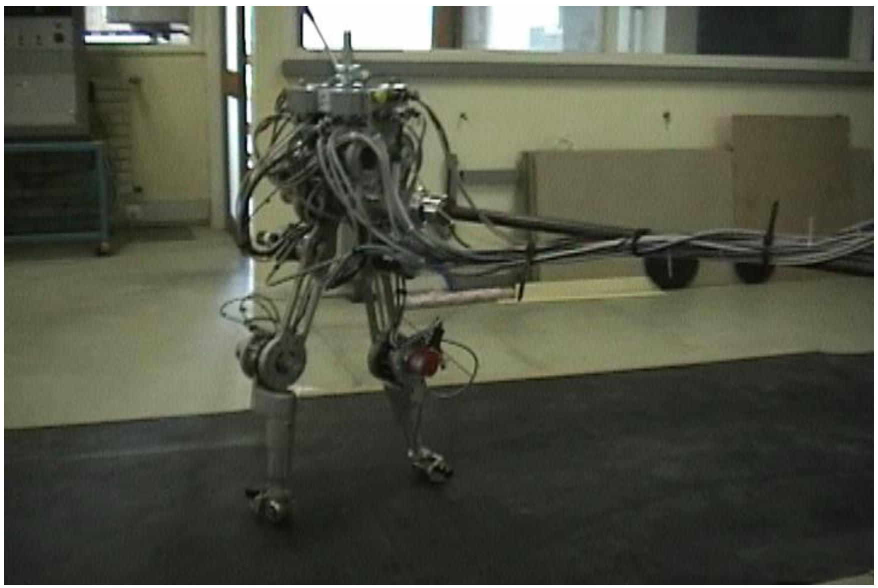


How to Use for Controller Design

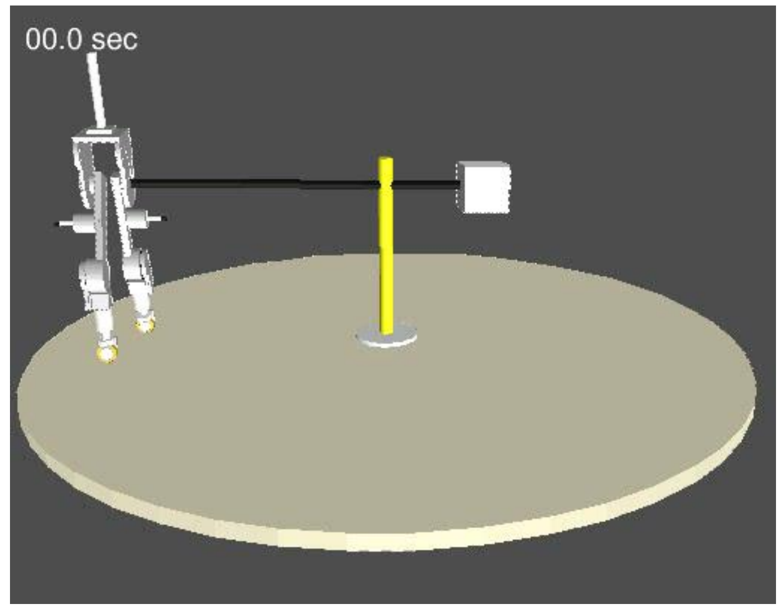
- Achieve performance by tuning parameters via optimization on 2-dimensional model, subject to previous constraints.

$$\begin{aligned} \dot{z} &= f_{a,zero}(z), & z^- &\notin S \cap Z_a \\ z^+ &= \Delta(z^-), & z^- &\in S \cap Z_a \end{aligned}$$

$$J(a) := \frac{1}{p_2^h(T^-)} \int_0^{T^-} \left\| u_a^*(t) \right\|^2 dt$$



LAG: Laboratoire Automatique de Grenoble



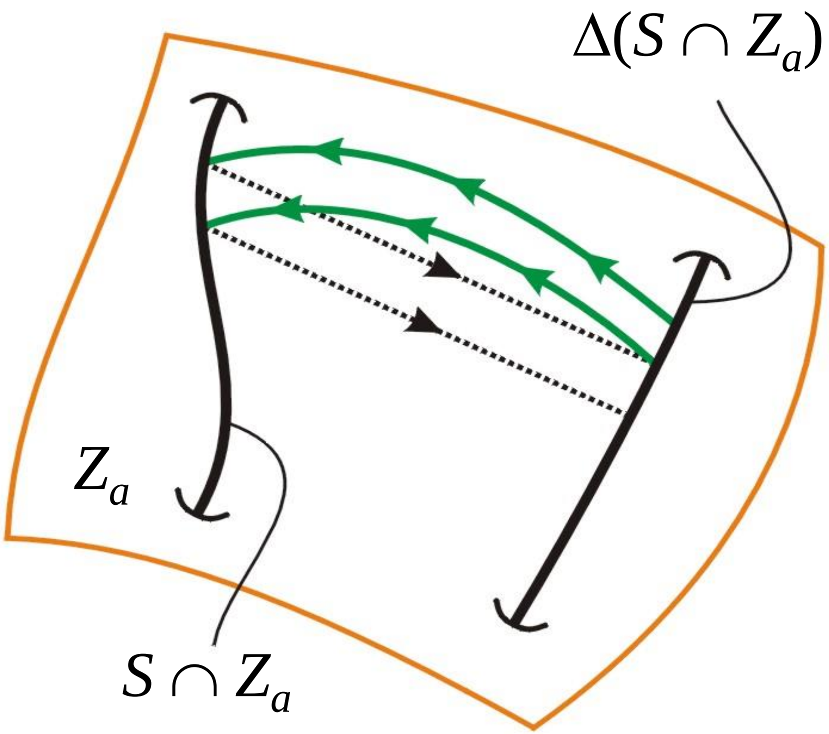
GeomView Animation by Evan Leung

Robustness Experiment

Robot + Controller = Time-Invariant, Hybrid, Exp. Stable, Oscillator!

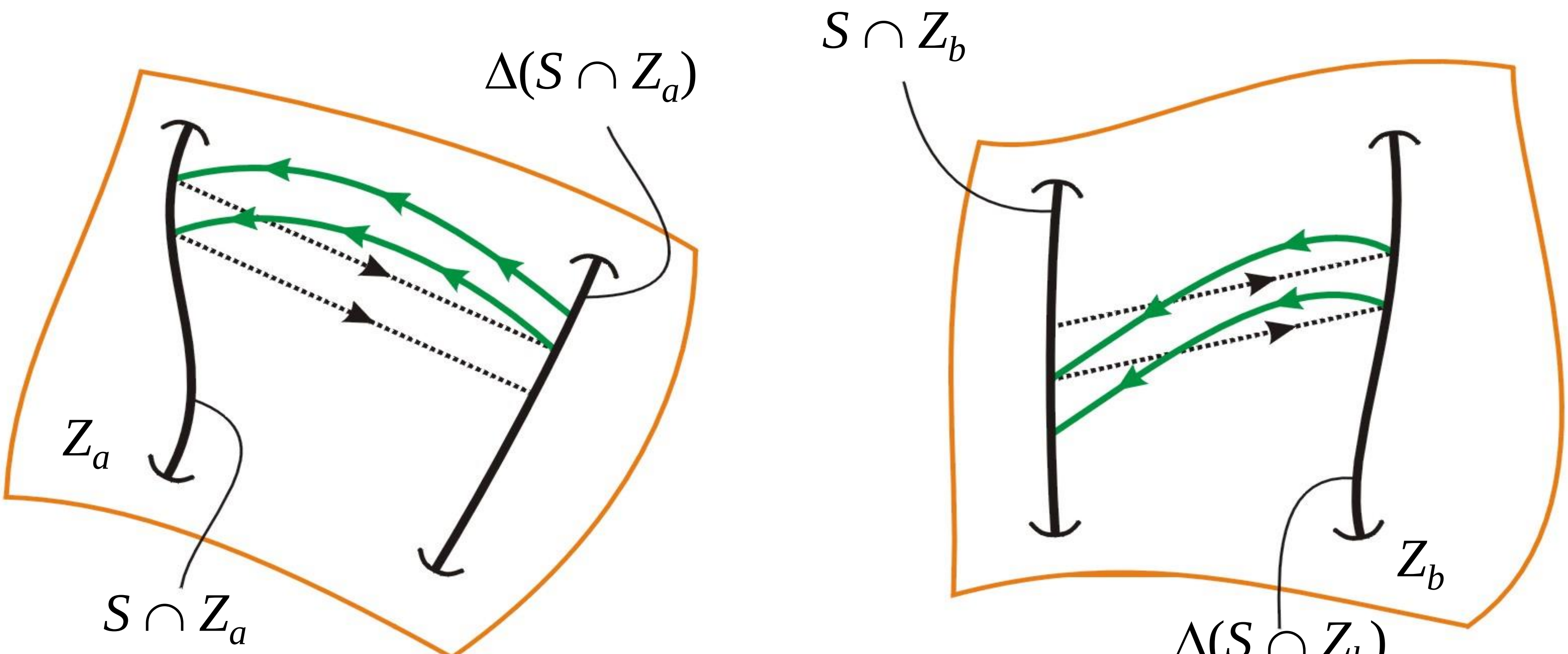
Composition of Walking Motions

- Introduce controller to transition from domain of one Poincaré map to another



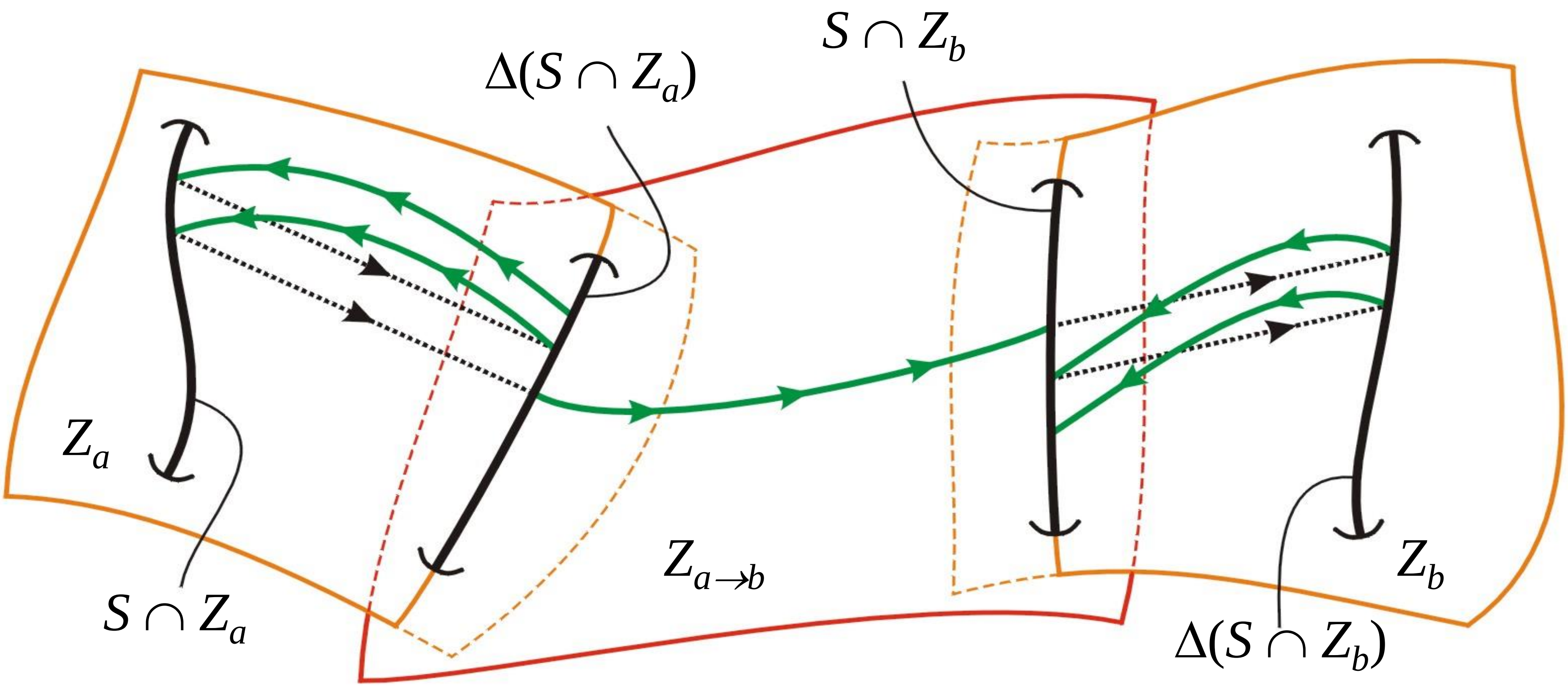
Composition of Walking Motions

- Introduce controller to transition from domain of one Poincaré map to another



Composition of Walking Motions

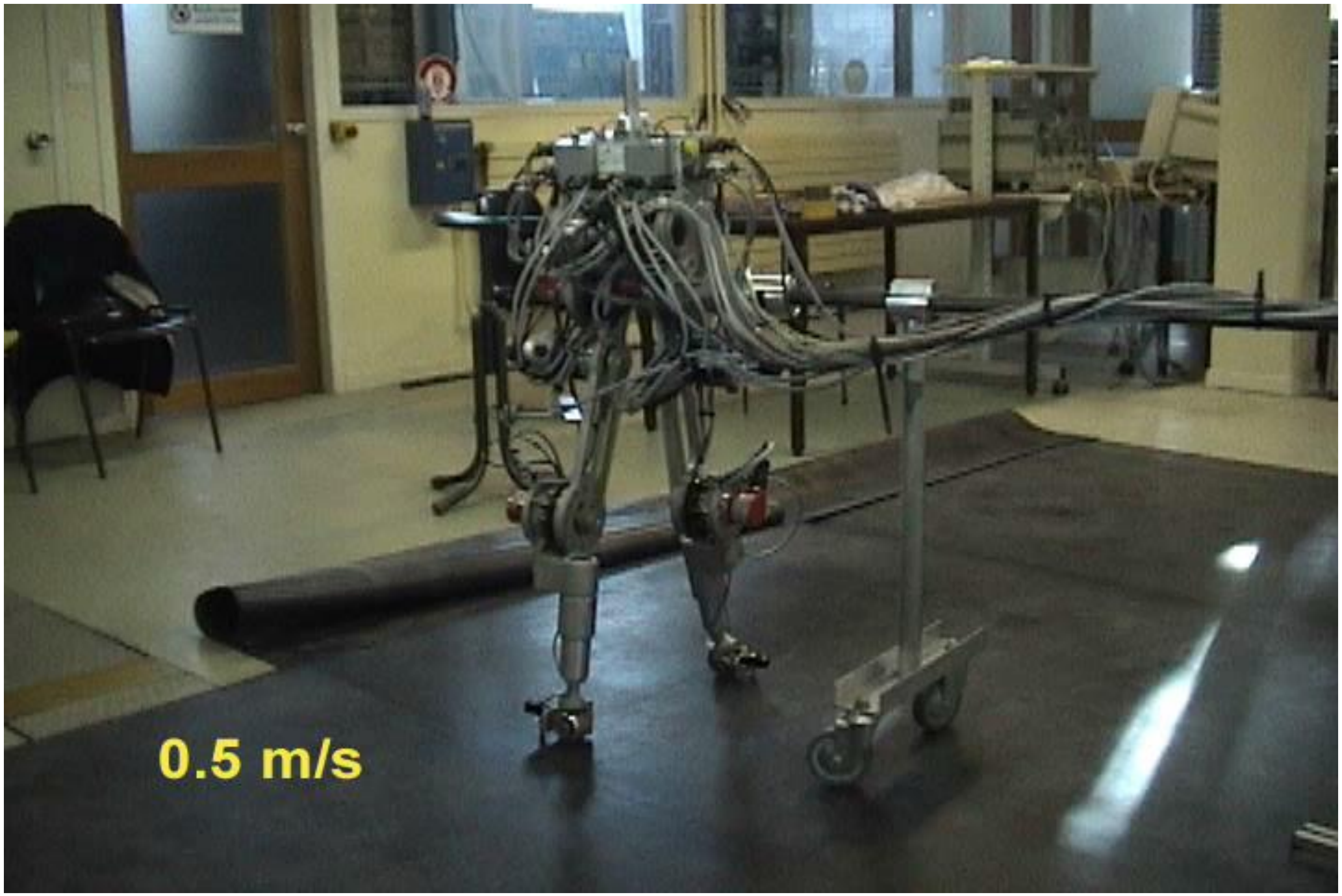
- Introduce controller to transition from domain of one Poincaré map to another



Westervelt, Grizzle, & Canudas-de-Wit (2003)

Experimental Implementation

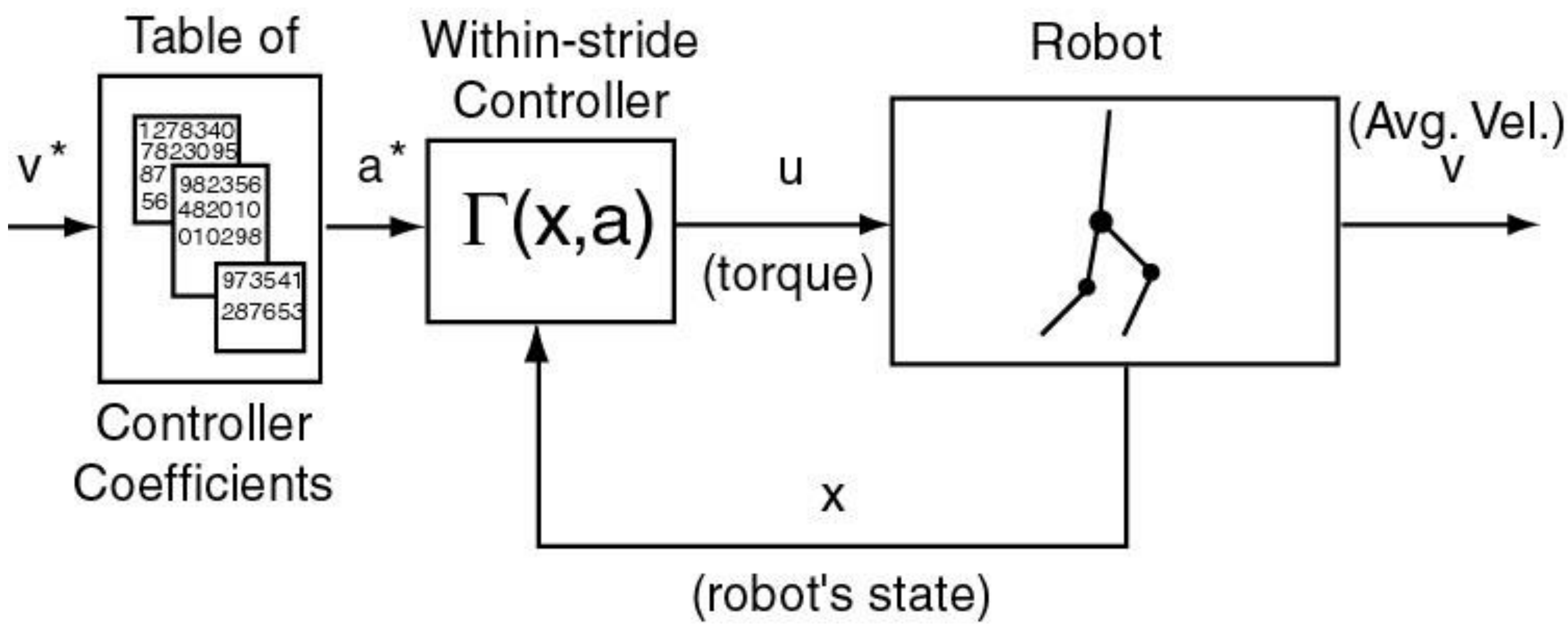
$0.5 \rightarrow 0.6 \rightarrow 0.7 \rightarrow 0.8 \rightarrow 0.7 \rightarrow 0.6 \rightarrow 0.5 \rightarrow \ldots$



LAG: Laboratoire Automatique de Grenoble

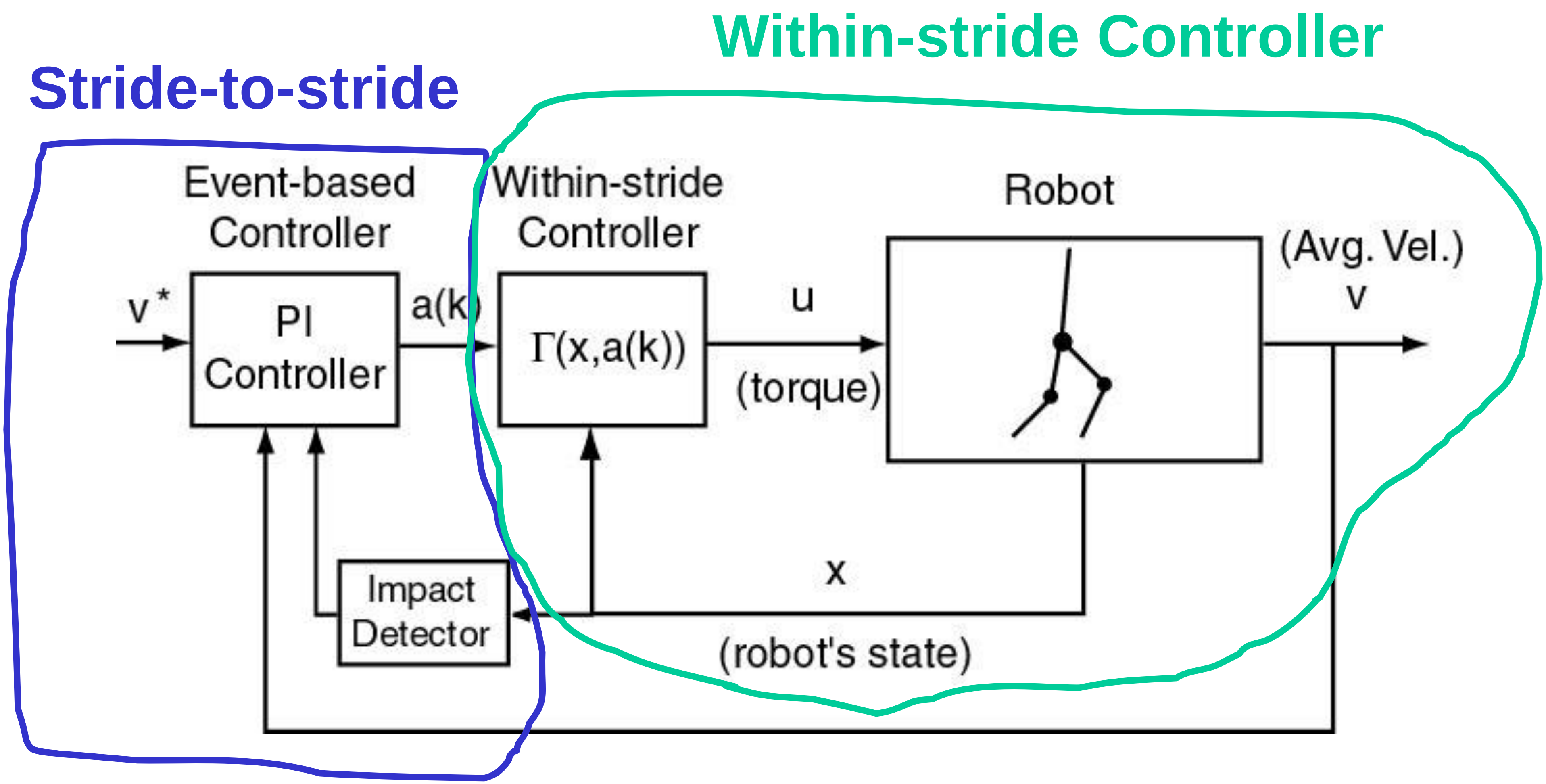
Event-Based Control

Key Idea: Use the parameters of the within-stride controller as control knobs



Event-Based PI Control

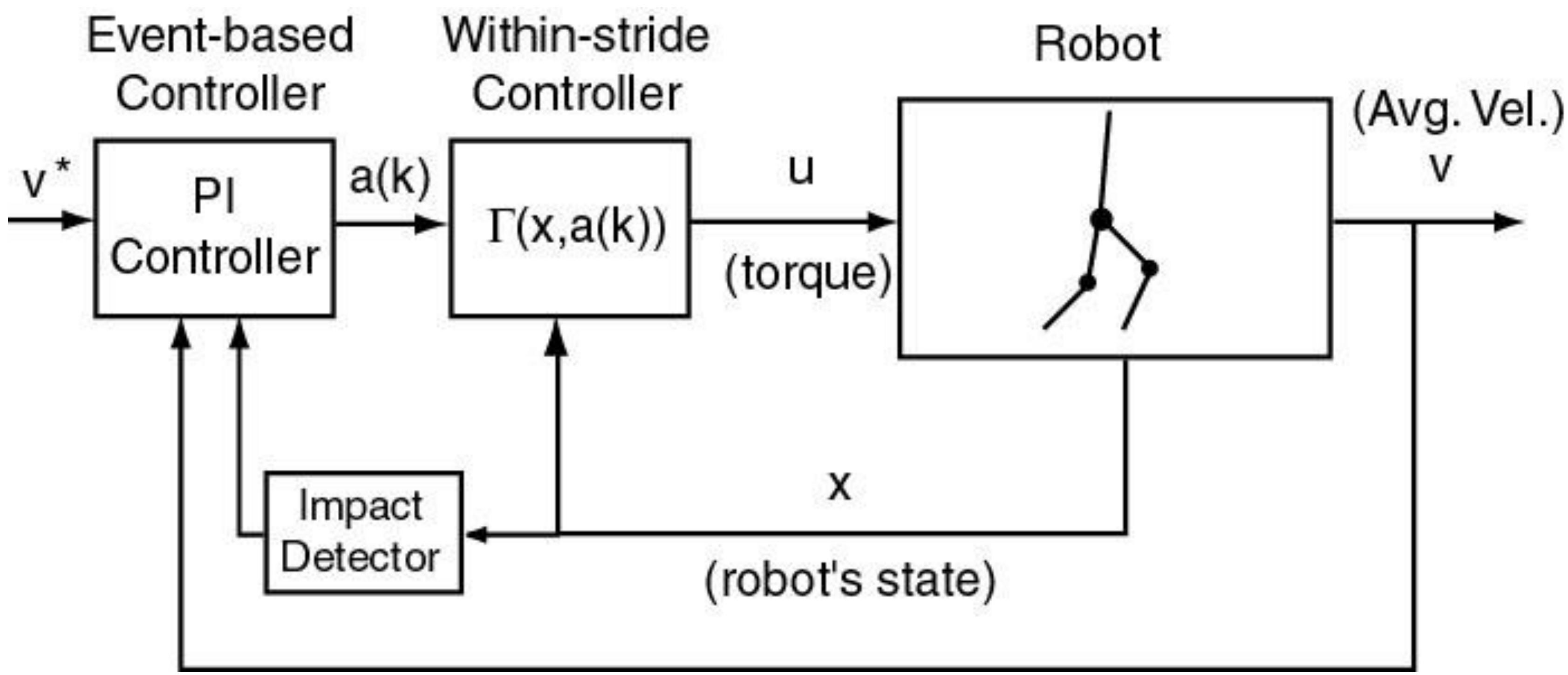
Key Idea: Use the parameters of the within-stride controller as control knobs



Event-Based PI Control

Key Idea: Use the parameters of the within-step controller as control knobs

- Maintain invariance
- Modify "posture (surface)" to change speed



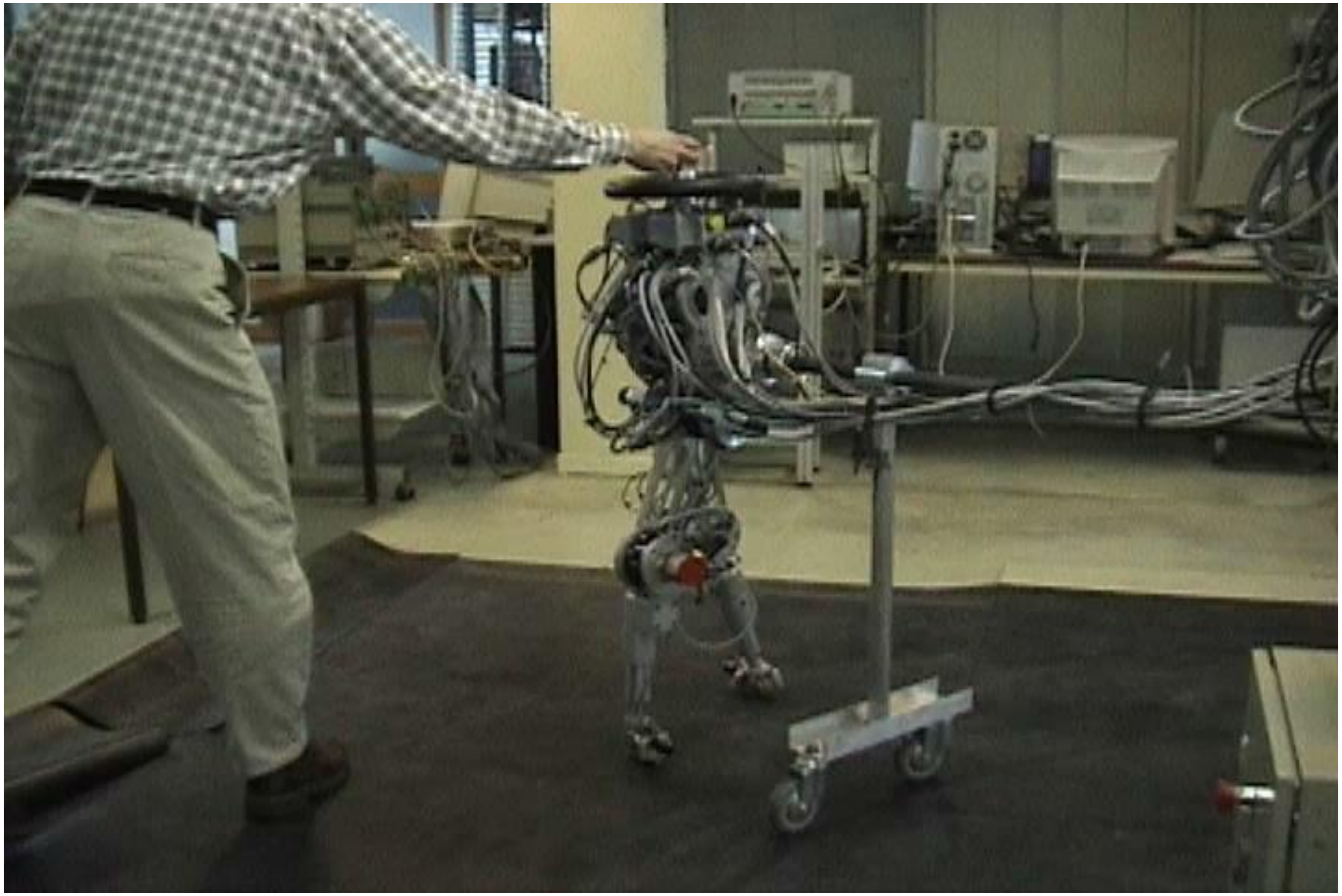
Experimental Implementation

(PI control to reject perturbation)

Extra mass shifts fixed point to faster walking speed

PI ON

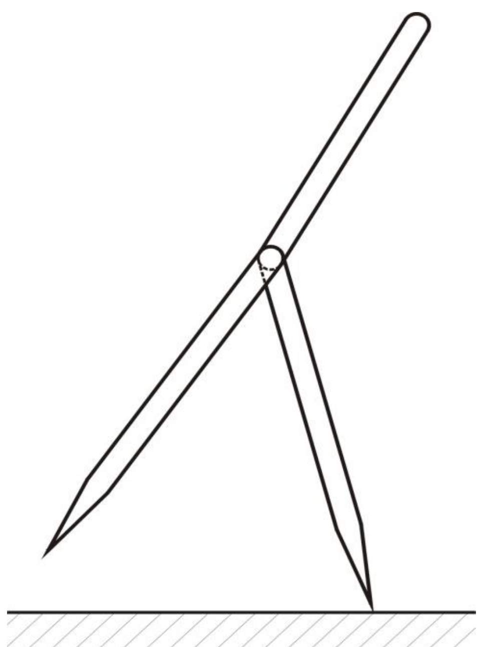
Event based control recovers original walking speed



Westervelt, Grizzle, & Canudas-de-Wit (2003)

Natural Progression

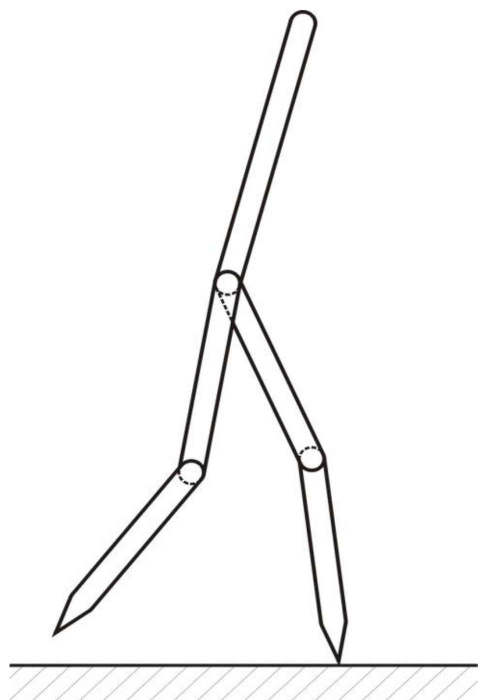
Grizzle, Abba & Plestan 1999



→

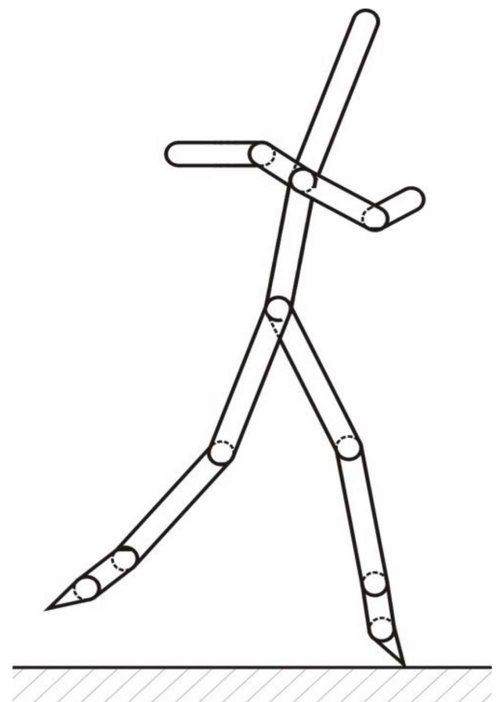
RABBIT

Plestan, Grizzle, Abba & Westervelt 2000



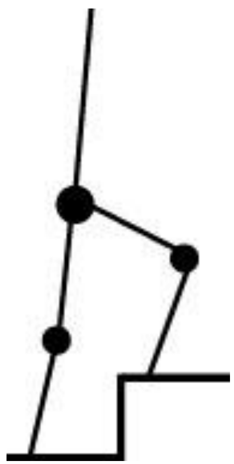
→

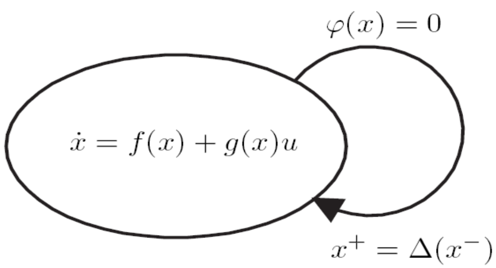
Westervelt, Grizzle & Koditschek 2001



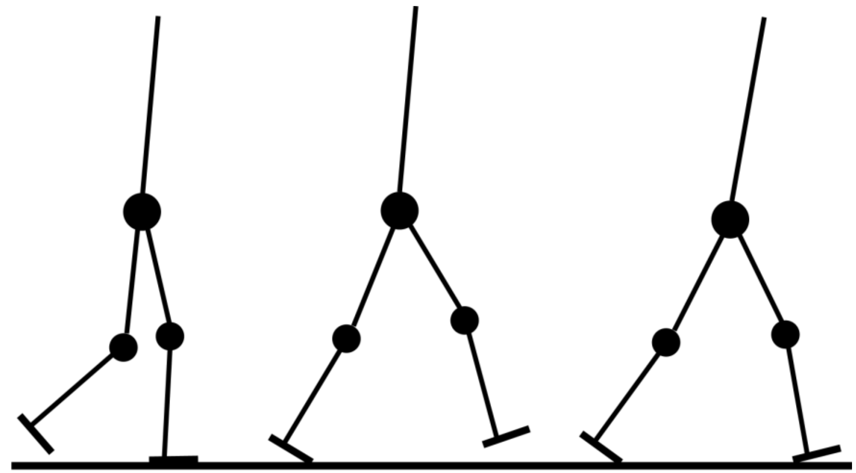
Natural Extensions

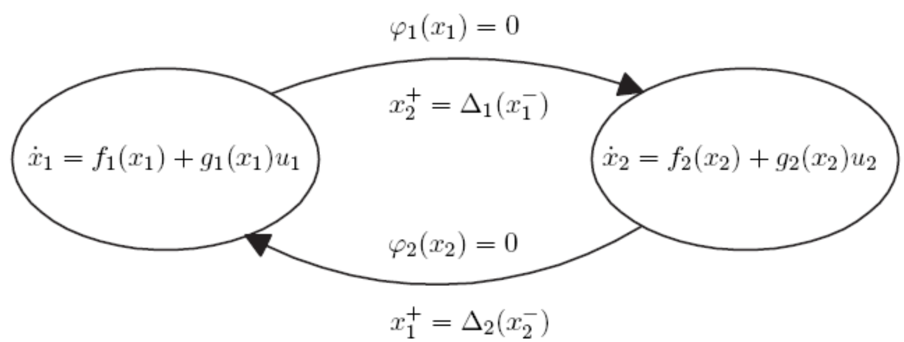
Stairs or Slopes





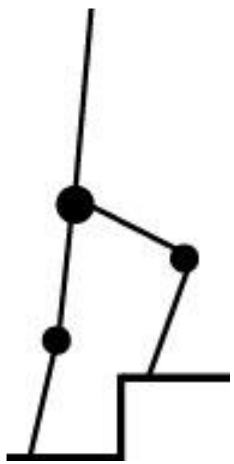
Adding Feet





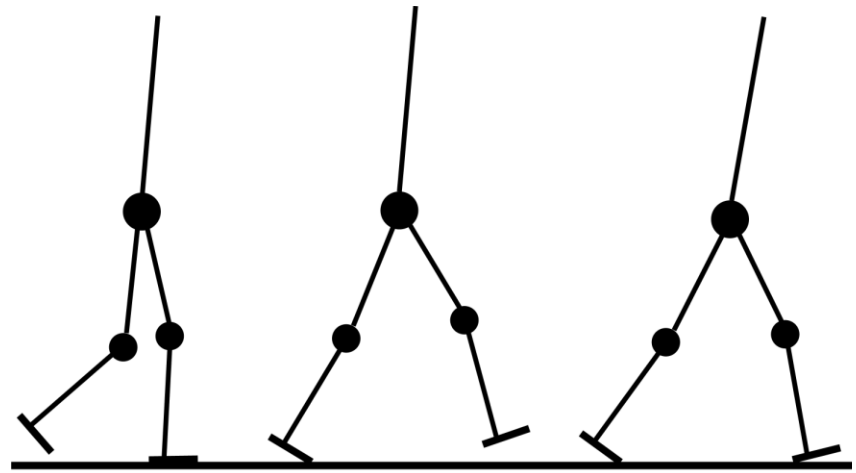
Natural Extensions

Stairs or Slopes



- **Single Continuous-Phase**
  - underactuated
- Ben Morris, M.S. Work

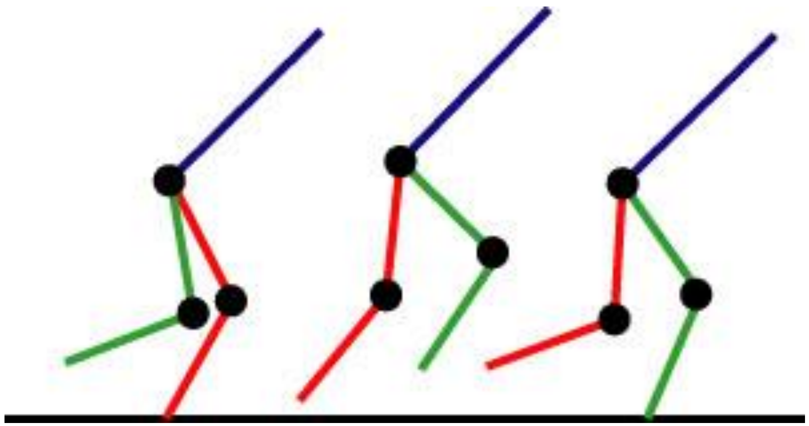
Adding Feet



- **Multiple Continuous-Phases**
  - fully-actuated
  - underactuated
  - over-actuated
- Jun Ho Choi, ACC-2005 (submitted)

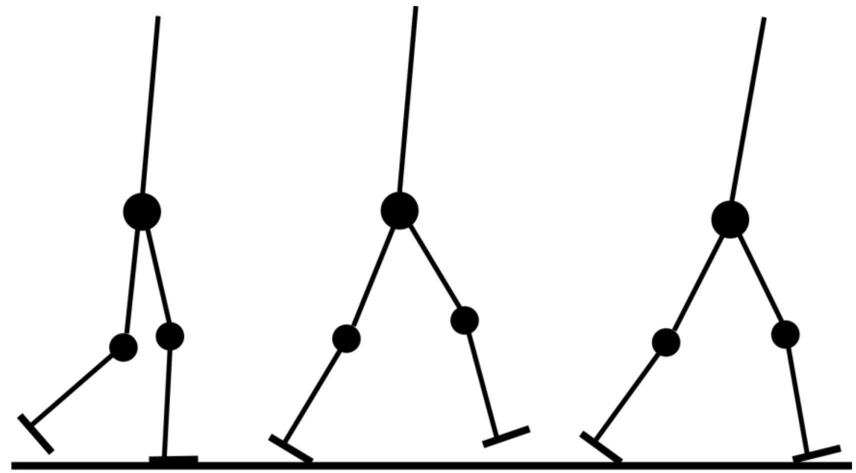
Natural Extensions

Running



- **Multiple Continuous-Phases**
  - single support
  - flight
  - varying degree of actuation
- CDC 2004

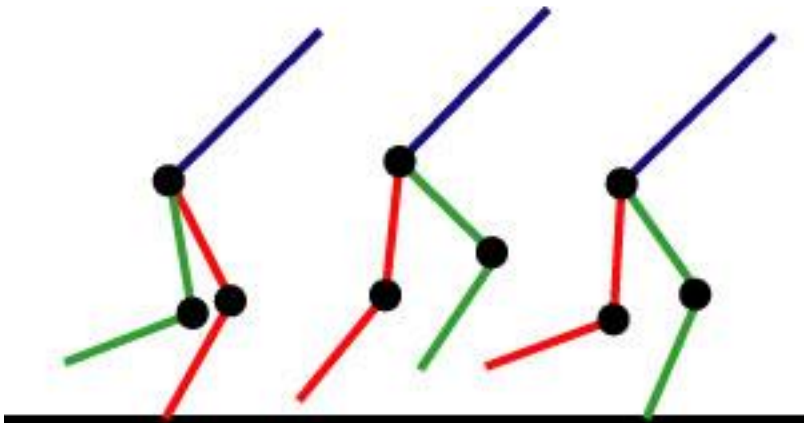
Adding Feet



- **Multiple Continuous-Phases**
  - fully-actuated
  - underactuated
  - over-actuated
- Jun Ho Choi, ACC-2005 (submitted)

Natural Extensions

Running



- **Multiple Continuous-Phases**
  - single support
  - flight
  - varying degree of actuation
- CDC 2004 (Chevallereau, Westervelt)

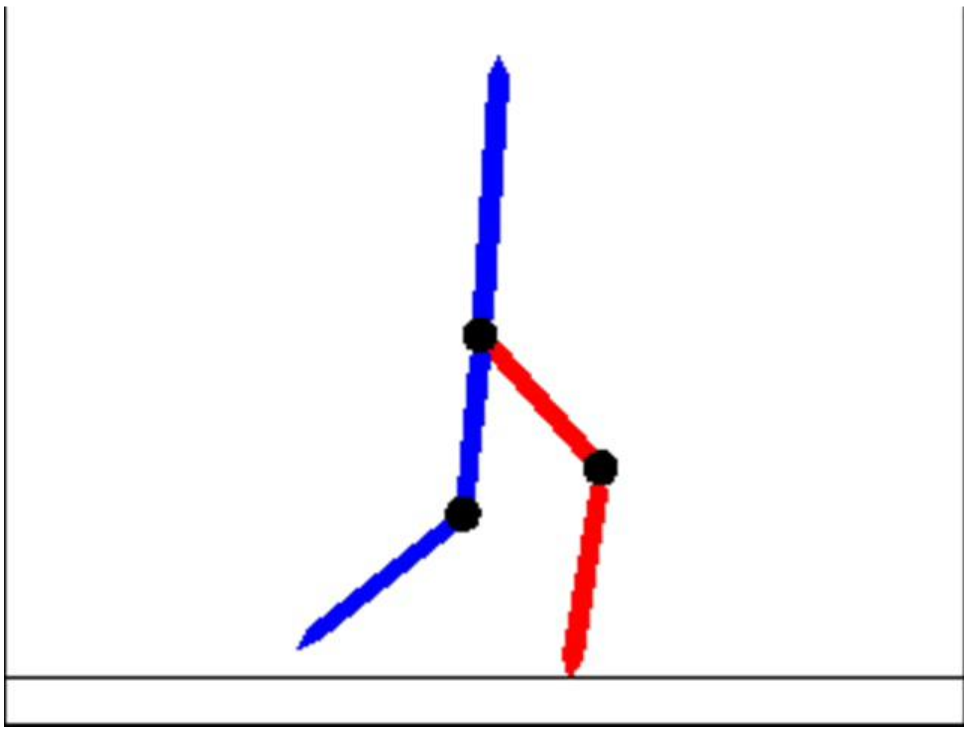
- Theory parallels HZD of walking
- Novel part: event-based control of the flight phase
- Closed-form computation of reduced Poincaré map
- Experiments started…

B. Morris
C. Chevallereau

G. Buche
E. Westervelt

Natural Extensions

Running



- Theory parallels HZD of walking
- Novel part: event-based control of the flight phase
- Closed-form computation of reduced Poincaré map
- Experiments started…

B. Morris
C. Chevallereau

G. Buche
E. Westervelt

Six Steps Toward Infinity

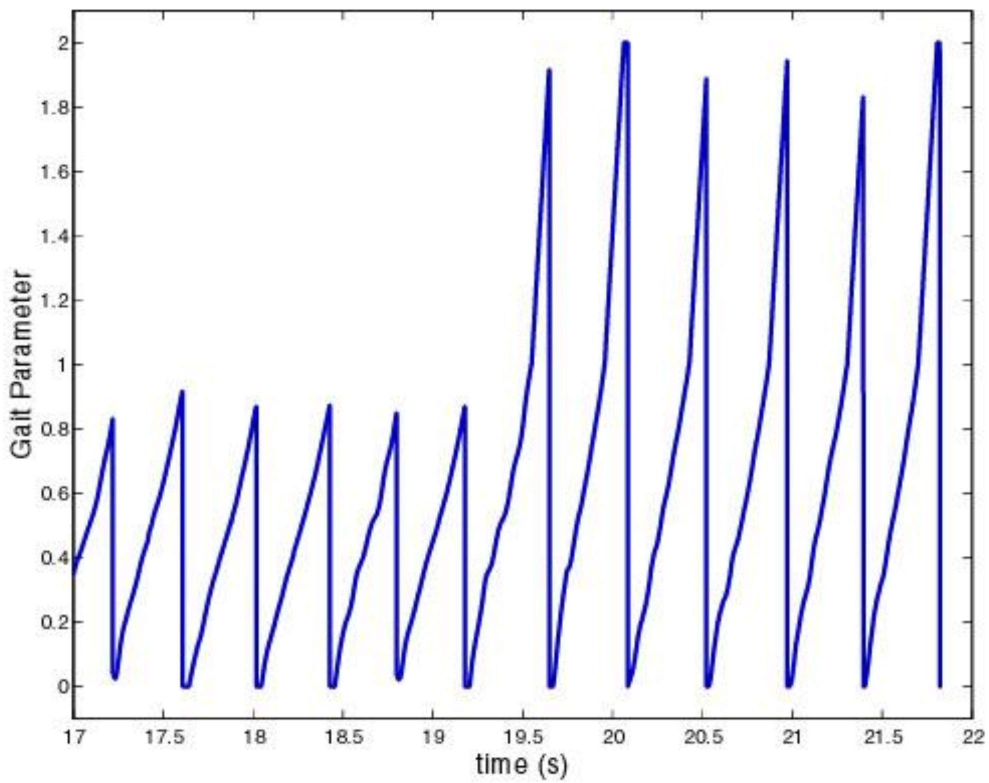
Our First Running Experiment

(September 2004)

Six Steps Toward Infinity

Our First Running Experiment

(September 2004)



Walking

Running!

<u>Score</u>

Sony ∞

Rabbit 6

Power Automatically Cut ☹

Many Open Problems

- Running ….Experimental verification!
- Controlled compliance (equivalent of tendons, muscles, …)
  - Energy efficiency
  - Impact attenuation
- Higher degrees of underactuation
- Remove the boom: 3-dimensional (non-planar) robots!
  [preliminary result: Doi, Hasegawa, & Fukuda, Humanoid Robots Conf., Oct. 2004]
- Much more is unknown than is known… rough terrain, vision, reflexes, …

Conclusions

- Models for legged robots are hybrid (ODE + Impact Map).
- Control strategy should be tailored to assist analysis and design
  - Hybrid zero dynamics
  - High analytical insight follows from low-dimensional geometry.
- Robot + Controller is a stable, time-invariant, hybrid, oscillator.
- Experiments are hard…but informative and exciting.
- Fortunately, we had time to think before experimenting.

Robot at Michigan

- Stay tuned! A robot is being designed.
- Joint with A. Rizzi & J. Hurst (CMU).

Your Logo Here!