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Practical Observers for Unmeasured States in Turbocharged Gasoline Engines

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Abstract— **Turbocharged gasoline engines are becoming more common in production vehicles as consumers demand improved fuel economy with uncompromised performance. Controlling this complex system to meet these and other competing objectives is a challenging task. Knowledge of exhaust manifold pressure and turbocharger speed can be important to success. Physical conditions of the system, however, make measurement impractical and costly, compelling manufacturers to implement some form of on-line estimation. Additional constraints imposed by computational resources and calibration processes limit application of a traditional state observer. In this paper, singular perturbation concepts are employed in concert with the reduced order observer to develop more practical estimates of exhaust manifold pressure and turbocharger speed. Simulation results show excellent observer performance with a significant reduction in calibration complexity.**

I. INTRODUCTION

Turbocharged gasoline engines are becoming more and more common in the marketplace due to the promise of improved fuel economy at equivalent performance. It is challenging for automakers to meet fuel economy, emissions, driveability and performance objectives while keeping cost affordable to consumers. They must rely heavily on the control strategy to deliver the expected benefits of this complex system.

Knowledge of current system behavior can be very beneficial for control. However, the turbocharged engine is a harsh environment for sensors, particularly when the variable of interest is in the exhaust path where temperatures can be extremely high. Therefore it is not always possible to reliably and/or affordably measure desired variables. This paper focuses on estimating typically unmeasured characteristics of the turbocharged gasoline engine: exhaust manifold pressure and turbocharger speed.

An estimate of exhaust manifold pressure (p_e) may be useful for many reasons, but it is particularly important for its role in emissions control. Emission control systems for gasoline engines rely heavily on feedforward air-fuel ratio (A/F) control to meet regulations. Since fuel flow rate can be regulated much more quickly than air flow rate, the feedforward fuel command is slaved to an estimate of the mass of air that will be in the cylinder at the time of combustion.[1] Stringent emissions standards and a slow acting feedback controller[2] dictate a very accurate air charge estimate.

Air charge is affected by exhaust manifold pressure when the intake and exhaust valve openings overlap. Fresh air in the cylinder is displaced when a pressure differential exists between the intake and exhaust manifolds during the overlap period. In turbocharged engines p_e can be quite large and variable. Therefore knowledge of p_e is helpful in meeting air charge accuracy requirements for turbocharged systems [1]. Static methods compensated with a simple lead filter are explored in [2], but a more accurate dynamic estimate is needed to drive the fast acting fuel delivery system.

An estimate of turbocharger speed (N_{tc}) may be useful for the torque delivery problem. The authors of [3] compared full state feedback control with conventional decentralized PI control using only pressure measurements. They found that state feedback led to equivalent response with significantly reduced actuator effort. Detailed analysis showed that N_{tc} feedback played an important role in the strategy's success. Therefore, we pursue a practical estimate of N_{tc}.

A traditional linear reduced order observer that estimates both N_{tc} and p_e immediately comes to mind. This approach, however, is not desirable for practical implementation due to calibration complexity and limited computing resources. In this paper, the estimation problems are separated and two observers are derived based on reduced order models developed using concepts from singular perturbation theory. This approach leads to fewer calibration parameters and facilitates implementation at different execution rates. This last point is particularly important since fast fuel delivery dictates that air charge estimates be computed as fast as every millisecond. With separate observers, the estimate of N_{tc} used for torque control can be easily implemented at a slower execution rate, thereby reducing the computational burden of the estimation task.

The paper is organized as follows. Section II briefly describes the mechanical system and presents the linear model used as the foundation for the work. In Section III, a traditional reduced order observer is developed as a benchmark for comparison for the simplified observers presented in Section IV. The paper concludes in Section V with a summary of results and a discussion of future work.

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[1]This mass of air is commonly referred to as air charge.

[2]Transport delays due to the location of the oxygen sensor(s) in the exhaust path limit the performance of feedback control.

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II. SYSTEM DESCRIPTION

The system under consideration is a turbocharged gasoline engine equipped with a conventional pneumatically operated wastegate to control boost. A schematic of the system is shown in Figure 1.

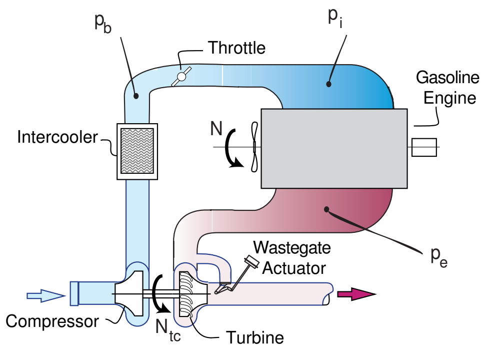

Fig. 1. Schematic of a turbocharged gasoline engine.

A nonlinear model describing such a system is presented in [3]. This model structure is augmented to include the nonlinear relationship between the control command, u_{wg}, and the flow rate through the wastegate, W_{wg}. A wastegate flow model similar in concept to that described in [4] is employed such that

$$W_{wg} = c_0(u_{wg}) + c_1(u_{wg})W_e, \quad (1)$$

where W_e is the total exhaust flow rate and c_0 and c_1 are defined as polynomial functions of u_{wg}.

With this addition, the model of the turbocharged gasoline engine takes the form

$$\begin{aligned} \dot{x} &= f(x,u,w) \quad (2)\\ x &= [p_i \;\; p_b \;\; p_e \;\; N_{tc}]^T \\ u &= [\theta \;\; u_{wg}]^T \\ w &= N \\ y &= [p_i \;\; p_b]^T \\ z &= Tq_{bk}, \end{aligned}$$

where x denotes the states of the system, u the actuator commands, w the exogenous input, y the measured outputs and z the unmeasured performance variable. This system is illustrated pictorially, along with a feedback control architecture, in Figure 2. The physical variables described by this model are intake manifold pressure (p_i), boost pressure (p_b), exhaust manifold pressure (p_e), turbocharger shaft speed (N_{tc}) and brake torque delivered by the engine (Tq_{bk}). Engine speed (N) is measured and slow moving, therefore it is treated as an input. The actuator commands are throttle angle (θ), and wastegate command (u_{wg}).

A linear representation of (2) is given by

$$\begin{aligned} \delta\dot{x} &= A\delta x + B\delta v \\ \delta y &= C\delta x \quad (3)\\ v &= [u \;\; w]^T \end{aligned}$$

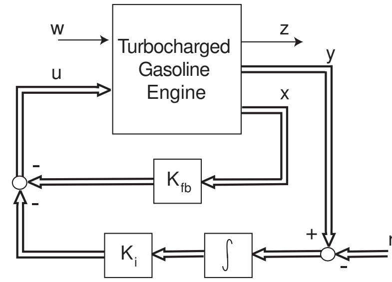

Fig. 2. System block diagram including a closed-loop control architecture.

where δ indicates deviation from the equilibrium point about which the system is linearized. A linearization around a typical operating point is given by

$$A = \begin{bmatrix} -25.23 & 10.97 & 4.36 & 0 \\ 0.89 & -105.26 & 0 & 147.67 \\ 208.47 & -8.20 & -204.6 & 9.29 \\ -2.68 & 56.65 & 7.40 & -86.52 \end{bmatrix},$$

$$B = \begin{bmatrix} 75.60 & 0 & -0.47 \\ -23.79 & 0 & 0 \\ 0 & 41.66 & 31.51 \\ 0 & 0 & 0 \end{bmatrix},$$

$$C = [I_{2\times2} \;\; 0_{2\times2}],$$

where the system states, inputs and outputs have been scaled by their maximum values.[3] This linearization is used to develop observer concepts and motivate singular perturbation arguments in Sections III and IV.

III. BENCHMARK: THE TRADITIONAL OBSERVER

The system (3) has four states, two of which are measured, p_i and p_b. The remaining two states, N_{tc} and p_e, can be estimated using a traditional reduced order linear observer. This approach is pursued to provide a benchmark to evaluate performance of more practical observers in Section IV.

Following the approach described in [5], the system is partitioned as follows

$$\begin{aligned} x_1 &= [p_i \;\; p_b]^T \\ x_2 &= [p_e \;\; N_{tc}]^T, \end{aligned}$$

such that

$$\begin{aligned} \delta\dot{x} &= \begin{bmatrix} \delta\dot{x}_1 \\ \delta\dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \delta v \\ \delta y &= \delta x_1 = [I_{2x2} \;\; 0_{2x2}] \begin{bmatrix} \delta x_1 \\ \delta x_2 \end{bmatrix}. \end{aligned}$$

A state description of the reduced order observer is given by

$$\begin{aligned} \dot{\zeta} &= A_r\zeta + B_r\beta \\ \hat{x}_2 &= C_r\zeta + D_r\beta \end{aligned}$$

[3]For example, when $\delta p_i = 0.1$ intake manifold pressure is perturbed by $+10\%$ of the maximum expected value, which in this case is assumed to be 200kPa.

where ζ is the observer state and

$$\begin{aligned}\beta &= [u \;\; y]^T \\ A_r &= [A_{22} - LA_{12}] \\ B_r &= [(B_2 - LB_1) \;\; (A_{22} - LA_{12})\, L + (A_{21} - LA_{11})] \\ C_r &= [I_{2x2}] \\ D_r &= [0_{2x2} \;\; L]\,.\end{aligned}$$

The observer gain matrix L is chosen such that $(A_{22} - LA_{12})$ is Hurwitz and the eigenvalues of this matrix are faster than the eigenvalues of A_{22}.

The performance of this 2^{nd} order observer is evaluated using simulation of the nonlinear model. Rather than choose arbitrary inputs, a state feedback controller is developed with integral action applied to the output errors to provide more realistic actuator commands for evaluation. The control architecture is depicted in Figure 2, where r refers to the reference commands for the outputs, p_i and p_b.

Linear quadratic regulator (LQR) methodology is employed to choose controller gains, using the engine torque response as the measure of success. The objectives are fastest possible rise time with minimal overshoot and no undershoot. For the linearization given in Section II, the state feedback gains, K_{fb}, and integral gains, K_i,

$$K_{fb} = \begin{bmatrix} 3.01 & 0.26 & 0 & 0.31 \\ 0.14 & 0.43 & 0 & 0.73 \end{bmatrix}, K_i = \begin{bmatrix} 22.19 & 1.10 \\ -0.87 & 2.81 \end{bmatrix},$$

produce the actuator commands and sensor outputs shown in Figure 3. These data will be used to drive each observer so that performance can be compared without the effects of compensation by the feedback control.

Performance of the benchmark observer in response to these inputs is shown in Figures 4 and 5. The estimates of p_e and N_{tc} are compared with simulated signals, represented by the solid lines in the figures. The observer performs quite well. The estimate of N_{tc} has a peak relative error[4] of approximately -0.04%, while the p_e estimate has relative error of less than 0.22%.

The reduced order observer is effective but it has 2 states and 18 parameters that must be calibrated,[5] many of which must be scheduled with operating condition for this nonlinear system. Although more complicated than desired for implementation, it serves as a good standard for comparison.

IV. SIMPLIFIED OBSERVERS

The approach to simplification exploits singular perturbation concepts to reduce the order of the linear model such that two first order observers can be used to produce separate estimates of N_{tc} and p_e.

The approach is motivated by taking a closer look at the linearized system (3). The eigenvalues of A are

$$\lambda_1 = -214.7, \quad \lambda_2 = -182.0,$$
$$\lambda_3 = -23.36, \quad \lambda_4 = -1.55,$$

[4]Relative error is defined as $100\,(x - \hat{x})\,/x$.

[5]Each matrix element is a calibration parameter.

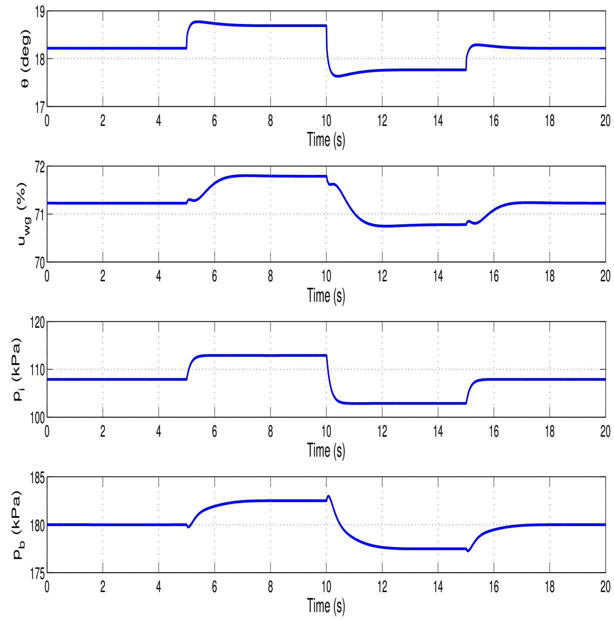

Fig. 3. Actuator inputs and sensor outputs used for transient simulation of estimators.

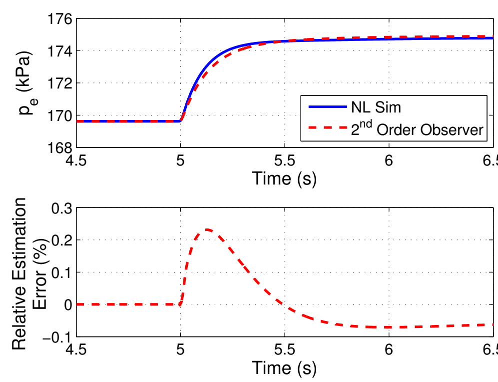

Fig. 4. Transient prediction of p_e using a traditional reduced order observer.

with corresponding eigenvectors,

$$\begin{aligned}e_1 &= \begin{bmatrix} -0.0334 & 0.1903 & 0.9710 & -0.1408 \end{bmatrix}^T \\ e_2 &= \begin{bmatrix} -0.0289 & 0.6718 & -0.6528 & -0.3489 \end{bmatrix}^T \\ e_3 &= \begin{bmatrix} 0.6396 & -0.1847 & 0.7386 & -0.1063 \end{bmatrix}^T \\ e_4 &= \begin{bmatrix} 0.3887 & 0.6829 & 0.3934 & 0.4773 \end{bmatrix}^T.\end{aligned}$$

This is clearly a multiple time scale system, with an order of magnitude separating λ_1 and λ_2 from λ_3 and λ_3 from λ_4. Therefore, model reduction using singular perturbation concepts seems plausible.

Since the eigenvalues of the system are real, it is straightforward to diagonalize the system by expressing it in the basis of its eigenvectors. This leads to a clear partitioning of the system where the three fastest states are grouped to form

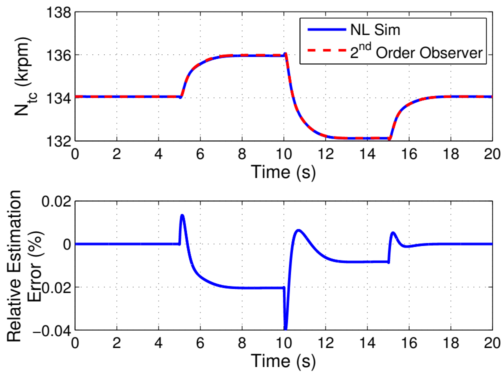

Fig. 5. Transient prediction of N_{tc} using a traditional reduced order observer.

the *fast* model and the slowest state defines the *slow* model. By assuming that the fast states change instantaneously from the viewpoint of the slow state, the *slow* model can be used to construct a first order observer for both p_e and N_{tc}. With this approach, the number of calibration parameters is reduced substantially to 14. The estimates are easily separated for implementation at different execution rates, but 10 parameters are required to estimate either p_e or N_{tc} alone. The possibility of further reduction in calibration complexity is explored with a different partitioning.

Ideally, the system would be partitioned using the physical variables, as this will keep physical intuition intact and avoid the computations that come with transformations. To this end, it is clearly seen that the first eigenvalue, λ_1, corresponds to the state p_e since the dominant component of e_1 is in the direction of the third state. Association of the remaining three eigenvalues to physical variables is not evident from the eigenvectors. From physics, however, it is known that the turbocharger speed responds on the order of λ_4 due to its large inertia and intake manifold pressure responds on the order of λ_3 due to the pumping action of the engine and its relatively small volume. This would imply that λ_2 corresponds to p_b.

At first glance, this doesn't make sense physically since the boost volume, which extends from the compressor outlet to the throttle inlet, is significantly larger than the intake manifold. Closer examination of the model, however, reveals that the response to throttle is governed by very fast dynamics. Consider the special case where the throttle is controlled perfectly to maintain a constant intake manifold pressure and turbocharger speed changes relatively slowly so it can be assumed to be constant. Then throttle is effectively a disturbance to the boost volume and

$$\delta \dot{p}_b = \frac{RT_b}{V_b}\left(\left(\frac{\partial W_c}{\partial p_b} - \frac{\partial W_{thr}}{\partial p_b}\right)p_b + \frac{\partial W_{thr}}{\partial \theta}\theta\right),$$

where R is the ideal gas constant for air, T_b is the boost volume temperature, V_b is the effective manifold volume, W_c is the flow out of the compressor into the volume and W_{thr} is the flow out of the volume through the throttle. Taking the Laplace Transform,

$$\frac{p_b(s)}{\theta(s)} = \frac{\frac{\partial W_{thr}}{\partial \theta}}{s + \frac{RT_b}{V_b}\left(\frac{\partial W_c}{\partial p_b} - \frac{\partial W_{thr}}{\partial p_b}\right)}.$$

For the equilibrium point defining the linear system (3), this first order transfer function has a time constant of 9.5ms, whereas intuition based on the volume alone leads to an expectation of 100ms or larger. At typical operating conditions, however, $\frac{\partial W_c}{\partial p_b}$ is quite large. This characteristic can be observed in the flat turbocharger speed lines on the left side of a typical compressor map, shown in Figure 6. As a result, this term dominates the expression and lowers the time constant relative to intuition.

This knowledge does not lead directly to separation of time scales of the physical variables since compressor flow rate also influences the behavior of N_{tc}. Nonetheless, we assign each eigenvalue to a physical state, accepting the error this may introduce. Specifically,

$$\lambda_1 : p_e, \quad \lambda_2 : p_b, \quad \lambda_3 : p_i, \quad \lambda_4 : N_{tc}.$$

These assignments are used to partition the system and develop reduced order observers for both N_{tc} and p_e.

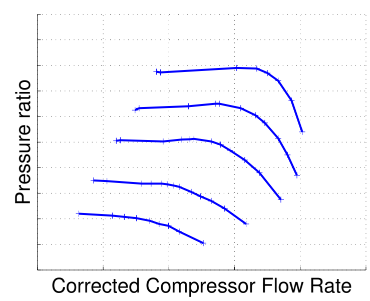

Fig. 6. Typical compressor map.

A. Observer for Turbocharger Speed

First consider turbocharger speed. The linear system model is partitioned as follows

$$\begin{aligned} x_s &= [p_i \;\; N_{tc}]^T \\ x_f &= [p_b \;\; p_e]^T \end{aligned} \tag{4}$$

such that

$$\begin{aligned} \delta\dot{x} &= \begin{bmatrix} \delta\dot{x}_s \\ \delta\dot{x}_f \end{bmatrix} = \begin{bmatrix} A_{s,1} & A_{s,2} \\ A_{f,1} & A_{f,2} \end{bmatrix}\begin{bmatrix} \delta x_s \\ \delta x_f \end{bmatrix} + \begin{bmatrix} B_s \\ B_f \end{bmatrix}\delta v \\ \delta y &= \delta x_s = [C_s \;\; C_f]\begin{bmatrix} \delta x_s \\ \delta x_f \end{bmatrix}. \end{aligned}$$

As suggested by singular perturbation theory [6], assume that the fast variables change instantaneously from the viewpoint of the slow states,

$$\begin{aligned} \delta x_f &= A_{f,2}^{-1}\left(A_{f,1}\delta x_s - B_2\delta v\right) \\ \delta\dot{x}_s &= \left[A_{s,1} - A_{s,2}A_{f,2}^{-1}A_{f,1}\right]\delta x_s \\ &\quad + \left[B_s - A_{s,2}A_{f,2}^{-1}B_f\right]\delta v \end{aligned}$$

By partitioning this system as follows

$$x_s = [x_{s,1} \;\; x_{s,2}]^T = [p_i \;\; N_{tc}]^T ,$$

the approach used in Section III can be applied to develop a reduced order observer for N_{tc}. A transfer function realization of the resulting observer is shown in Figure 7.

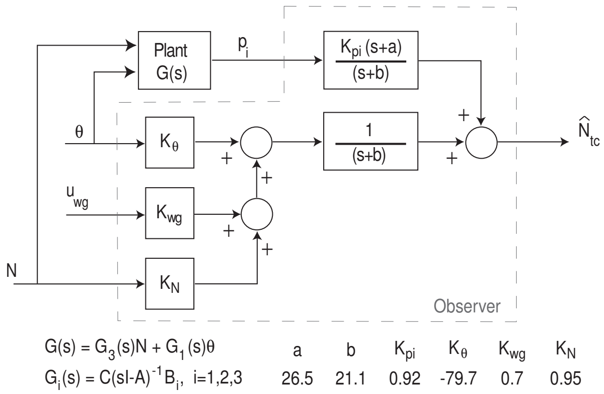

Fig. 7. Transfer function representation of first order observer for N_{tc}.

The response of this observer is compared with the benchmark and the actual simulated response in Figure 8. Observer performance is degraded relative to the benchmark but estimation error remains small. The number of calibration parameters to estimate N_{tc} with this approach is 6.

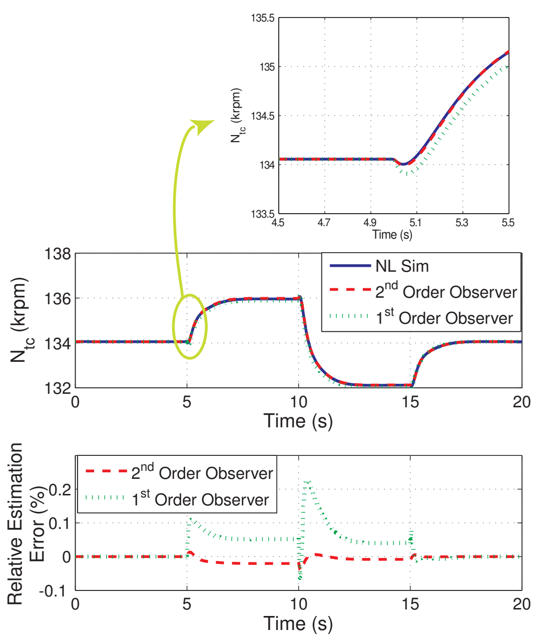

Fig. 8. Transient prediction of N_{tc} using a first order observer derived using singular perturbation concepts.

The number of calibration parameters has been reduced but by examining the system and observer in more detail it is possible to simplify even further. In this system, turbocharger speed can exhibit non-minimum phase behavior in response to throttle commands, as seen when zooming into the initial part of the transient in Figure 8. This behavior occurs when the power consumed by the compressor changes more rapidly than the power generated by the turbine. The compressor and turbine are both highly nonlinear devices and the difference between compressor power consumption and turbine power generation depends on the operating point and subtle phasing of the actuator commands. Uncertainties in the system due to manufacturing tolerances and aging may make it difficult to accurately predict this delicate balance over the life of the vehicle.

Consider how this non-minimum phase behavior is introduced in the observer by examining the transfer functions in Figure 7. Since all of the inputs are filtered at the same frequency, the gains of these first order transfer functions serve to define the steady state value of the estimate. The phasing of inputs, and therefore the non-minimum phase behavior, is introduced with the lag filter on the p_i input.

Since this lag is introduced at a frequency significantly faster than the bandwidth of the closed loop system (≈ 6 rad/s), it can be concluded that the non-minimum phase dynamics of turbocharger speed are not influential in the feedback system. In addition, if the observer gain is chosen such that the pole b is reasonably close to the zero a, no artificial input phasing is introduced when the lag filter is removed. Therefore, the lag filter can be eliminated and the number of calibration parameters needed to estimate N_{tc} is reduced to 5.

Figures 9 and 10 show that indeed the simplified observer response displays minimum phase behavior, but otherwise reflects the response of the 1^{st} order observer. The impact of this simplification on the closed loop system is demonstrated with simulation results in Figure 10, where the simplified observer produces the estimate of N_{tc} used by the controller. The pressure responses are virtually unchanged from the benchmark.

B. Observer for Exhaust Manifold Pressure

Now consider estimation of exhaust manifold pressure. The *fast* model as defined in (4) cannot be used to develop a reduced order observer since $(A_{f,2}, C_f)$ is unobservable,

$$\text{rank}\begin{bmatrix} C_f \\ C_f A_{f,2} \end{bmatrix} = \text{rank}\begin{bmatrix} 1.0 & 0 \\ -105.26 & 0 \end{bmatrix} \neq 2.$$

Instead p_i is grouped with the fast variables to achieve an observable system and the approach described in Section III is applied with the estimate of turbocharger speed as an input.

The resulting observer gain corresponding to the p_b measurement is 0. Therefore the system is simplified by partitioning as follows,

$$x_\phi = [x_{\phi,1} \;\; x_{\phi,2}]^T = [p_i \;\; p_e]^T .$$

Then

$$\delta \dot{x}_\phi \;=\; A_{\phi,2}\delta x_\phi + [B_\phi \; A_{\phi,1}] \begin{bmatrix} \delta v \\ \delta \hat{N}_{tc} \end{bmatrix},$$

where $A_{\phi,i}$, $i = 1, 2$, and B_ϕ are the appropriate matrix entries from system (3).

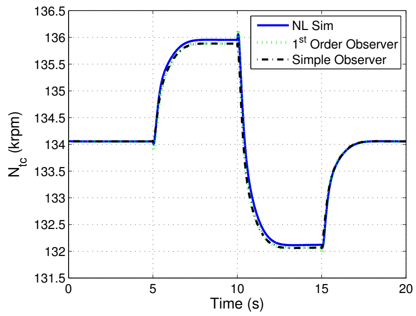

Fig. 9. Transient prediction of N_{tc} using a simplified first order observer.

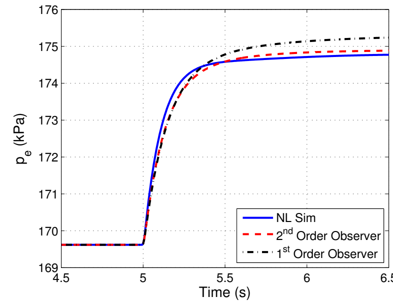

Fig. 11. Transient prediction of p_e using a first order observer derived using singular perturbation concepts.

The response of the reduced order observer developed from this simplified system is compared to responses of the nonlinear simulation and the 2^{nd} order benchmark observer in Figure 11. Peak relative estimation error is similar to the benchmark but the steady state error is larger. Overall, estimation error remains less than 0.27%. The number of calibration parameters required to formulate this estimate is 8 but since this observer requires knowledge of the estimated slow variable N_{tc}, the total number of calibration parameters is 14. The total number of calibration parameters required to estimate both states has not been reduced beyond that achieved with diagonalization. This partitioning, however, offers an alternative separation of the estimates which may be more computationally efficient, depending on the software architecture.

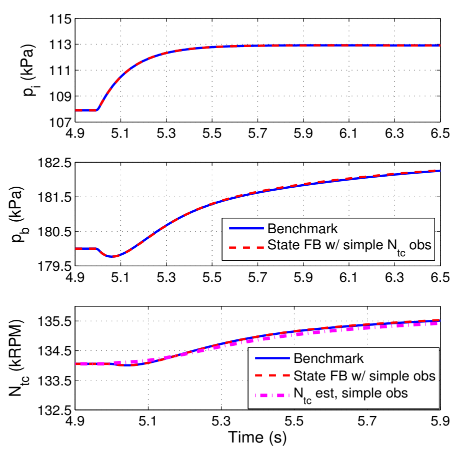

Fig. 10. Nonlinear system response with state feedback and simplified N_{tc} observer.

V. CONCLUSIONS AND FUTURE WORK

Singular perturbation concepts have been used to develop first order observers for turbocharger speed and exhaust manifold pressure. Model reduction of a diagonalized system yields a single observer that estimates both states and reduces the number of calibration parameters by more than 20% relative to the benchmark. Implementation of the estimates at different execution rates is straightforward, but 10 calibration parameters are required to estimate only one of the states.

An alternative system partitioning results in an observer for N_{tc} that requires only 6 calibration parameters. This observer captures the non-minimum phase behavior of N_{tc} in response to a throttle input via lag compensation of the intake manifold pressure measurement. In cases where the knowledge of the non-minimum phase behavior is not critical, it has been shown that the number of calibration parameters can be reduced to five by removing this lag effect.

Using this estimate of turbocharger speed as an input to an observer for p_e requires 8 additional calibration parameters. The total number of calibration parameters required to estimate both states is unchanged from the diagonalized system. This approach, however, offers an alternative for implementation at different execution rates.

Plans for future work include a robustness assessment, as well as a study to determine a straightforward approach for scheduling the calibration parameters with operating condition. Finally, the concept will be validated with engine dynamometer and/or vehicle data.

VI. ACKNOWLEDGMENTS

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